Physics 121MA/Math 122PH
Fall 2002 Project 4 Errata

Problem 1(b) should read: Derive an upper bound on the error for the midpoint approximation Mid_n in terms of a, b, n, and M_2 where M_2 is an upper bound for $|f''(x)|$.

Here's how to get started: The difference between the function $f(x)$ and the linear approximation $l(x)$ based at $\frac{c+d}{2}$ has an upper bound given by

$$\big|f(x) - l(x)\big| \leq \frac{1}{2}M_2\Big(x - \frac{c+d}{2}\Big)^2.$$

This is just Equation (8.12) on page 286 with some different notation.

The error in the midpoint (= "tangent line at the midpoint" approximation) for the subinterval $[c, d]$ is given by

$$\left|\int_c^d \big(f(x) - l(x)\big)\, dx\right| \leq \int_c^d \big|f(x) - l(x)\big|\, dx \leq \int_c^d \frac{1}{2}M_2\Big(x - \frac{c+d}{2}\Big)^2\, dx.$$

The last integral in the line above can be evaluated exactly. The result can be expressed in terms of a, b, n, and M_2 using

$$d - c = \Delta x = \frac{b-a}{n}.$$

This result thus applies to each subinterval so a bound on the total error is given by n times this result.

When the dust settles, the error bound you get can be compared with the error bound we know for the original trapezoid approximation, namely

$$\frac{1}{12}M_2\frac{(b-a)^3}{n^2}.$$

The relationship between the two error bounds should be obvious.