

**Instructions:** We encourage you to work with others in your assigned group on this project. You should write your solution neatly using complete sentences that incorporate all symbolic mathematical expressions into the grammatical structure. Include enough detail to allow a fellow student to reconstruct your work, but you need not show every algebraic or arithmetic step. It is important that you do your own writing, even if you have worked out the details with other people. All graphs should be done carefully on graph paper or drawn by a computer. This project is due at the beginning of class on Wednesday, September 11.

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1. Nature has some odd secrets. Among them are the fundamental constants, facts about nature that we can measure but cannot account for with any theory. Three of these constants are the speed of light,  $c = 3.00 \times 10^8$  m/s, the universal gravitation constant,  $G = 6.67 \times 10^{-11}$  m<sup>3</sup>/kg-s<sup>2</sup>, and the fundamental constant of quantum physics known as Planck's constant,  $h = 6.63 \times 10^{-34}$ kg m<sup>2</sup>/s.

Suppose a physicist wanted to estimate the size of the universe at an early time, when relativity, quantum mechanics and gravitation all came into play. The answer to this physics problem must involve the fundamental constants – what else? Without even knowing how to set up this physics problem, one can still use dimensional analysis to find a quantity with the dimensions of length using these constants. To proceed, we can write the so-called *Planck length* as

$$L_p = c^i G^j h^k$$

where  $i$ ,  $j$ , and  $k$  need not be integers, but are chosen so that the right hand side has the dimensions of length.

- (a) Determine  $i$ ,  $j$ , and  $k$  and calculate the Planck length. Big or small? Compared to what?
  - (b) Repeat the process to find a quantity with the dimensions of time. This is called the *Planck time*, and gives an estimate of the age of the observable universe when it was the size of the Planck length.
  - (c) Repeat the process to find a quantity with the dimensions of mass. This is called the *Planck mass*, an estimate of the mass of the universe at the Planck time.
2. (a) Is there a smallest positive number? Is there a smallest positive rational number? Is there a smallest positive irrational number? Give an argument to support your answer for each of these questions.
    - (b) Does your calculator “think” there is a smallest positive number? Give an argument to support your answer. Include some information on the calculator model you use.
    - (c) Write  $0.\bar{9}$  as a ratio of integers (that is, as a rational number). Comment on your intuition for the result. Can you find a way to think about this so the result “feels” right?