Physics 121MA/Math 122PH

Fall 2001 Project 7 Name ________________________

Instructions: We encourage you to work with others in your assigned group on this project. You should write your solution neatly using complete sentences that incorporate all symbolic mathematical expressions into the grammatical structure. Include enough detail to allow a fellow student to reconstruct your work, but you need not show every algebraic or arithmetic step. It is important that you do your own writing, even if you have worked out the details with other people. All graphs should be done carefully on graph paper or drawn by a computer. This project is due at the beginning of class on Tuesday, November 20.

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1. In this problem you will compute the rotational inertia of a uniform solid disk with total mass M and radius R. Let the symbol σ represent the mass density, which in this context means mass per unit area.

   (a) Find an expression for the total mass M in terms of σ and R.

   (b) Consider a thin ring of radius r ($r < R$) centered at the disk's center. Let Δr be the size of the ring in the radial direction. Compute the mass of the ring in terms of σ, r, and Δr.

   (c) Letting your answer for (b) be m_i, put that answer into the definition of rotational inertia

   $$I = \sum_{i=1}^{n} m_i r_i^2$$

   so that I is expressed as a Riemann sum.

   (d) Taking the limit as $n \to \infty$ (or $\Delta r \to 0$), express I as a definite integral.

   (e) Evaluate the definite integral to find the rotational inertia as a function of M and R (not σ). Your answer should be of the form $I = fMR^2$ where f is a fraction less than 1.

2. The *Koch snowflake sequence* is described in Problem 9 of Section 11.2. The elements in the sequence are polygons and the limit of the sequence is called the *Koch snowflake*. The point of Problem 9 is to show that the perimeter of the Koch snowflake is infinite. That is, the perimeters of the polygons in the sequence increase without bound as the sequence index increases without bound. Now focus on the region enclosed by the Koch snowflake. Construct an infinite series that gives the area of this region and find the sum of that infinite series.