Physics 121MA/Math 122PH
Fall 2001 Project 6 Name ______________________

Instructions: We encourage you to work with others in a small group on this project. You should write your solution neatly using complete sentences that incorporate all symbolic mathematical expressions into the grammatical structure. Include enough detail to allow a fellow student to reconstruct your work, but you need not show every algebraic or arithmetic step. It is important that you do your own writing, even if you have worked out the details with other people. All graphs should be done carefully on graph paper or drawn by a computer. This project is due at the beginning of class on Friday, November 2.

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1. Working in groups of up to four persons, make whatever measurements you want on the baseball bat (please no permanent marks or changes in the bat's structure!). Based on these measurements, come up with a function $\lambda(x)$ for the linear mass density (mass per unit length) of the bat. It is up to you to choose whether your function $\lambda(x)$ will be in the form of a table of numerical values *or* a symbolic function. In either case you should discuss the relative merits of these two types of functions for this specific problem. Using your function $\lambda(x)$, determine the bat's total mass and the position of the center of mass. Compare your model's results with the actual mass and center of mass.

2. The **midpoint approximation** of a definite integral $\int_a^b f(x)\ dx$ is the rectangular approximation in which the height of the rectangle on the ith subinterval is given by the output of the function for the midpoint of the subinterval. For the subinterval $[c, d]$, the area of the approximating rectangle is $f\left(\frac{c+d}{2}\right) \Delta x$.

   (a) Illustrate the midpoint approximation for a generic subinterval. Use pictures to argue that the area of the midpoint rectangle is equal to the area of a trapezoid in which the top of the trapezoid is given by the tangent line to the graph of f at the midpoint $\left(\frac{c+d}{2}, f\left(\frac{c+d}{2}\right)\right)$. (*Hint:* Think about rotating the top of the midpoint rectangle.)

   (b) Use pictures to compare the error in the original trapezoid approximation with the error in the midpoint approximation for a generic subinterval. For this it is useful to use the "new trapezoid" view of the midpoint approximation that is suggested by (a). Make a conjecture about the relative sign of the two errors and the relative magnitude of the two errors.

   (c) Compute the trapezoid approximation and the midpoint approximation (with the same number of subintervals) for a few examples for which you can also compute an exact value using the Second Fundamental Theorem. Use the exact value to compute the actual total errors. Compare these errors to the conjecture you made in (b).

   (d) Based on the evidence from (b) and (c), find the best combination of the trapezoid approximation and the midpoint approximation that is more accurate than either approximation alone for the same number of subintervals. If all has gone well, you should arrive at the approximation known as **Simpson's rule**.

   (e) Use Simpson's rule to approximate $\displaystyle\int_0^2 e^{-x^2}\, dx$ with $n = 5$ subintervals.