A Course and Text Integrating Calculus and Physics

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January 12, 1999

We are in the fifth year of teaching a year-long course that integrates calculus and physics. Our goal is to increase student understanding by synchronizing related topics in calculus and physics. The course is team-taught and meets eight hours per week. We are preparing a text to disseminate the curriculum we have developed.

Some issues to address

The main issue we seek to address is the transfer of knowledge between courses by students who take both calculus and physics. As specific barriers to this transfer we see a lack of synchronization in related topics along with differences in terminology, notation, and style. We address all of these in our course and text.

A student taking both calculus and physics will see many topics in both courses with mathematics being applied in the physic course and physics used as a source of motivation and applications in the calculus course. Rarely are these synchronized in separate courses. Worse, math faculty face constrains of time and expertise in fully developing applications while physics faculty face similar constrains in fully utilizing calculus. Many of us have heard anecdotes from our physics colleagues about asking students if they know a particular concept from calculus and getting a negative response. This is partly giving the safe response on the part of the students because the students know they will not be held accountable. It is also partly due to the differences in terminology, notation, and style.

One major lack of synchronization comes in the area of vectors and vector-output functions. Typically these are topics for the third semester of the calculus sequence but used

*The development of this course and text was supported by U.S. Department of Education FIPSE Grant No. P116B40024.

early in the first semester of the introductory physics course. Our course synchronizes these topics through two moves. First, we require the first semester of calculus as a prerequisite. Most of the students in our course are first year students who are ready to start the second semester of calculus after a year of calculus in high school. Second, we introduce vectors and vector-output functions at the beginning of the course. (Several other integrated programs, including those at Auburn University and Diablo Valley College, have similar early introduction of vector-output functions.) For our students the study of vector-output functions has an additional advantage of providing a new context in which to review the ideas of limit, continuity, and derivative.

We address the issue of differences in terminology, notation, and style throughout the course. As an example of a difference in terminology, physicists tend to use the term *integral* without the distinctions among *definite integral*, *antiderivative*, and *indefinite integral* made by mathematicians. As an example of a difference in notation, physicists use unit coordinate vectors (e.g. $2\hat{\imath} + 6\hat{\jmath} + 3\hat{k}$) almost exclusively while mathematicians often write a vector as an ordered triple (e.g., $\langle 2, 6, 3\rangle$). As an example of a difference in style, physicists often construct definite integrals using "infinitesimals" while mathematicians generally construct a Riemann sum and then invoke a limit process. Our strategy is not to choose one side or the other but instead to encourage fluency in both approaches. We are careful to point out the differences but we use just one choice in each problem we write so students must be able to understand both. We allow each student to write with his or her own choice of terminology and notation.

Structure of the integrated course

We are at a four year liberal arts institution with approximately 2700 undergraduates. There is no engineering program but we do have a strong dual degree program and many entering students think of themselves as potential engineers. A good number of those students do carry on in the dual degree program while others choose to pursue different majors including one of the sciences, mathematics, or computer science.

The physics department offers a year-long introductory physics sequence beginning each fall. Fall enrollment is typically 100 to 120 students in three sections (including the integrated section). The mathematics department offers a three semester calculus sequence with sections for all three courses offered each semester. The third semester covers multivariate material. Fall enrollment is typically about 180 for the first semester course, about 140 for the second semester course, and about 30 for the third semester course.

In the integrated course we combine material from the second and third semesters of the calculus sequence with material from the year-long physics sequence. As a prerequisite we assume each student has had the first semester of the calculus sequence or a year of high school calculus. Currently each student is registered for a particular section of the relevant calculus course (second semester in the fall and third semester in the spring) and a particular section of the relevant physics course. These sections are scheduled in consecutive hours so that we have two 50 minute blocks of class time four days each week. (Traditional math and physics courses meet four days per week.) Students also have an separate lab session each week.

Internally we treat the course as one class, a fact we make clear in all of our advertising and advising. Homework assignments, weekly projects, and exams are all integrated. This is not to say that every question is integrated but many are and we do not record separate scores for calculus material and physics material. In the end we give each student one grade and record that grade for both the calculus course and the physics course. On the student's transcript the two grades appear to be the same by coincidence.

On a typical day each instructor leads the class for one of the 50 minute blocks. When warranted one instructor may lead the class for more than half of the total time in order to complete a topic that will be used or amplified by the other instructor.

Our enrollment averages about 20 students in both fall and spring. However these are generally not the same 20 students. Typically we lose 2 or 3 students going from fall to spring while picking up about the same number. In several years the spring semester enrollment has included *all* students concurrently taking third semester calculus and second semester physics. In our experience and from discussions with colleagues we have not found difficulties with students moving in or out of the integrated sequence. There is some potential for more difficulty with this starting in the current year because we have begun use of our own text and made more radical changes in the ordering of material.

For the first four years of the course we used the same calculus and physics texts as all other sections of the relevant sequences. In using separate texts we were forced to rearrange the ordering of chapters which can create some problems for students and occasionally required supplementary material. Beginning this year we are using a draft of our own text. The text will be available from Addison Wesley Longman for the 1999-00 academic year. A table of contents is included as an appendix.

The table of contents gives a good guide to the course curriculum. The first two chapters contain a review of limit, continuity, and derivative for scalar-output functions and introduce these ideas for vector-output function. It is important to note that not all of the material is directly related between the two disciplines. At points each discipline pursues essentially unrelated topics while at other points the topics are intimately related.

The integrated course is established here at UPS. Several faculty from each discipline have taught the course and we have the support of both departments. Each department has at least three members interested in teaching the course. The course should continue indefinitely.