

Due April 18

Name

Directions: Be sure to include in-line citations, including page numbers if appropriate, every time you use the results of discussion, a text, notes, or technology. **Only write on one side of each page.**

“The road to wisdom? Well it plain and simple to express: Err and err and err again, but less and less and less.” -Piet Hein, poet and scientist (1905-1996)

Problems

1. Adapt Euclid’s proof of the infinitude of prime integers to show that for any field F , there are infinitely many monic irreducible polynomials in $F[x]$.
 - (a) Also explain why this argument fails for the formal power series ring $F[[x]]$.
2. Partial Fractions for polynomials
 - (a) Prove that every rational function in $\mathbf{C}[x]$ can be written as a sum of a polynomial and a linear combination of functions of the form $1/(x-a)^k$.
 - (b) Find a basis for $\mathbf{C}(x)$ as a vector space over \mathbf{C} .
3. Let a and b be relatively prime integers. Prove there are integers m, n such that $a^m + b^n \equiv 1 \pmod{ab}$