Mathematics 434 **Spring 2014** **Problem Sheet 2**

January 30, 2014

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Name

Directions: Be sure to include in-line citations, including page numbers if appropriate, every time you use the results of discussion, a text, notes, or technology. **Only write on one side of each page.**
"Mathematics consists of proving the most obvious thing in the least obvious way." – Polyá, George (1887, 1985)

Problems

1. If R is any ring, and the map $\phi : \mathbf{Z} \to R$ is defined by

$$\phi(n) = \begin{cases} 1_r + \cdots + 1_r, & n > 0 \text{ (where there are } n \text{ terms in the sum)} \\ -\phi(-n) & \text{if } n < 0 \end{cases}$$

   (a) Use the Peano axioms to show that this map is compatible with addition of positive integers. That is, $\phi(m+n) = \phi(m) + \phi(n)$ for all $m, n \in \mathbf{Z}$.

   (b) Use the facts that ϕ is compatible with addition and multiplication of positive integers to show that it is compatible with addition and multiplication of all integers.

2. Use the substitution principle to show that, for any ring R, $R[x,y] \approx R(x)[y]$. Hint:

   (a) Extend $R \to R[x][y]$ to a map Φ

   (b) Extend $R[x] \to R[x,y]$ to a map Ψ

   (c) Use uniqueness of extension to show $\Phi \circ \Psi$ and $\Psi \circ \Phi$ are both the identity maps.

   (d) Prove that if $f : A \to B$ and $g : B \to A$ are maps satisfying $f \circ g = id_B$, then f is surjective and g is injective and hence the map Φ in part c. is an isomorphism.