Mathematics 434 **Problem Sheet 1**

____
Name

Directions: Be sure to include in-line citations, including page numbers if appropriate, every time you use the results of discussion, a text, notes, or technology. **Only write on one side of each page.**
"Experience is what enables you to recognize a mistake when you make it again." (Earl Wilson)

Problems

1. Given a ring R, the set of formal power series $p(t) = a_0 + a_1 t + a_2 t^2 + \cdots +$ ('formal' means there is no requirement of convergence) is a ring. (This set is denoted $R[[t]]$.) Show that $R[[t]]$ is a ring under the standard addition and multiplication of power series and prove that a formal power series $p(t)$ is invertible in $R[[t]]$ if and only if a_0 is a unit of R.

2. Let $\mathbb{Q}$ denote the rational numbers (you may use the fact that $\mathbb{Q}$ is a field), $\mathbb{Q}[\alpha]$ the smallest subring of C (the complex numbers) containing $\mathbb{Q}$ and α, and $\mathbb{Q}[\alpha, \beta]$ the smallest subring of C containing $\mathbb{Q}$ and both α and β. Let $\alpha = \sqrt{2}$, $\beta = \sqrt{3}$ and $\gamma = \alpha + \beta$. Prove that $\mathbb{Q}[\alpha, \beta] = \mathbb{Q}[\gamma]$.

3. (Judson: Chapter 16, #6) Find all the ring homomorphisms $\phi : \mathbf{Z}/6\mathbf{Z} \to \mathbf{Z}/15\mathbf{Z}$.

4. Do both of the following:

   (a) Judson: Chapter 16, #36 (c),(d). You may assume parts (a) and (b) are true.

   (b) Let I and J be subrings of a ring R and define $I + J = \{a + b : a \in I \text{ and } b \in J\}$.

      i. Prove that $I + J$ need not be a subring of R.

      ii. Prove that if one of I and J, say J, is an ideal, then $I + J$ is a subring of R. (Hence the Second Isomorphism Theorem for Rings in Judson [Theorem 16.13] now makes sense.)