Smith **Math 290** **Spring 2014**

Proof SLE-2

Accepted **Not Accepted**

I affirm this work abides by the university's Academic Honesty Policy.

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Print Name, then Sign

- First due date **Tuesday, February** 18.
- Turn in your work on a separate sheet of paper with this page stapled in front.
- Do not include scratch work in your submission.
- There is to be **no collaboration** on any aspect of developing and presenting your proof. Your only resources are: you, the course textbook, me, and pertinent discussions that occur **during class.**
- Follow the Writing Guidelines of the Grading Rubric in the course information sheet.
- Retry: Only use material from the relevant section of the text or earlier.
- Retry: Start over using a new sheet of paper.
- Retry: Restaple with new attempts first and this page on top.

"*It is by logic that we prove but by intuition that we discover.*" (Henri Poincaré)

SLE-2 (**Use only material up to and including Section HSE**)
Let A be an $m \times n$ matrix, and $\vec{b}$ a constant vector for which the system of equations $LS\left(A, \vec{b}\right)$ is consistent and has solution set S. Pick one vector in S and denote it by $\vec{\beta} = \begin{bmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_n \end{bmatrix}$. Let T be the set of all vectors obtained by adding the components of $\vec{\beta}$ to the corresponding components of each of the vectors in $N(A)$, the null space of A. More specifically, $T = \left\{ \begin{bmatrix} a_1 + \beta_1 \\ a_2 + \beta_2 \\ \vdots \\ a_n + \beta_n \end{bmatrix} \in \mathbf{C}^n \;\middle|\; \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{bmatrix} \in N(A) \right\}$. Prove that the sets S and T are equal.

For all $m \times n$ matrices A and all vectors $\vec{b} \in \mathbb{C}^m$ for which the system of equations $LS\left(A, \vec{b}\right)$ is consistent with solution set S.

For all $\vec{\beta} = \begin{bmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_n \end{bmatrix} \in \mathbb{C}^n$,

prove, if $T = \left\{ \begin{bmatrix} a_1 + \beta_1 \\ a_2 + \beta_2 \\ \vdots \\ a_n + \beta_n \end{bmatrix} \in \mathbf{C}^n \;\middle|\; \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{bmatrix} \in N(A) \right\}$, then $S = T$.

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