

Differentiability

Math 280

Spring 2013

Idea of differentiability

For $f : \mathbb{R} \rightarrow \mathbb{R}$

- Idea: f is differentiable at x_0 if zooming in on the graph at $(x_0, f(x_0))$ gives a line
- Definition: f is differentiable at x_0 if $\lim_{h \rightarrow 0} \frac{f(x_0 + h) - f(x_0)}{h}$ exists

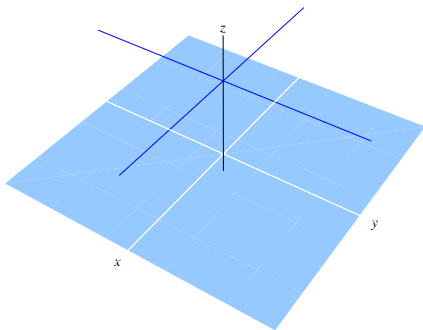
For $f : \mathbb{R}^2 \rightarrow \mathbb{R}$

- Idea: f is differentiable at (x_0, y_0) if zooming in on the graph at $(x_0, y_0, f(x_0, y_0))$ gives a plane
- Definition: ???

It is not enough to know that $f_x(x_0, y_0)$ and $f_y(x_0, y_0)$ exist.

Example: a nondifferentiable function

$$f(x, y) = \begin{cases} 1 & \text{if } x = 0 \text{ or } y = 0 \\ 0 & \text{otherwise} \end{cases}$$



Note that $f_x(0, 0) = 0$ and $f_y(0, 0) = 0$ so the partial derivatives exist for $(0, 0)$ *but* zooming in on $(0, 0, 1)$ does not give a plane.

A sufficient condition for differentiability

Defining **differentiable** for $f : \mathbb{R}^2 \rightarrow \mathbb{R}$ is a bit messy.

Can give a sufficient condition for differentiability:

Theorem:

If (x_0, y_0) is a point in the domain of a function f with

(A) f defined for all points in an open disk centered at (x_0, y_0) ,
and

(B) f_x and f_y each continuous for all points in that open disk

then f is differentiable for (x_0, y_0) .

Differentiability as a hypothesis for other results

Differentiability is often a condition needed as a hypothesis.

Theorem:

If f is differentiable in an open region containing the point P , then

$$\left(\frac{df}{ds} \right)_{P, \hat{u}} = \vec{\nabla} f(P) \cdot \hat{u}.$$