Math 280 | Multivariable Calculus | Spring 2013

3-D Basics

From *University Calculus; Hass, Weir, Thomas*

1. Give a geometric description of the set of points in space whose coordinates satisfy the given pairs of equations.
   (a) $x = 2,\ y = 3$
   (b) $y = 0,\ \ z = 0$
   (c) $x^2 + y^2 = 4,\ \ z = 0$
   (d) $x^2 + z^2 = 4,\ y = 0$
   (e) $x^2 + y^2 + z^2 = 1,\ x = 0$
   (f) $x^2 + y^2 + (z+3)^2 = 25,\ z = 0$

2. Describe the sets of points whose coordinates satisfy the given inequalities and equations.
   (a) $x \geq 0,\ y \geq 0,\ z = 0$
   (b) $x^2 + y^2 + z^2 > 1$
   (c) $x^2 + y^2 + z^2 \leq 1,\ z \geq 0$

3. Describe the given set with an equation or a pair of equations.
   (a) The plane perpendicular to the y-axis at $(0, -1, 4)$.
   (b) The plane through the point $(3, -1, 1)$ parallel to the xz-plane.
   (c) The circle of radius 2 centered at $(0, 2, 0)$ and lying in the yz-plane.
   (d) The line through the point $(1, 3, -1)$ parallel to the x-axis.
   (e) The circle in which the plane through the point $(1, 1, 3)$ perpendicular to the z-axis meets the sphere of radius 5 centered at the origin.
   (f) The set of points in space equidistant from the origin and the point $(0, 2, 0)$.

4. Write inequalities to describe the following sets.
   (a) The interior and exterior of the sphere of radius 1 centered at the point $(1, 1, 1)$.
   (b) The closed region bounded by the spheres of radius 1 and radius 2 centered at the origin. (**Closed** means the spheres are to be included. If we wished to leave the spheres out we would be describing the **open** region bounded by the spheres. This should remind you of the way we use **closed** and **open** to describe intervals: closed sets include boundaries; open sets leave them out.)

5. Find the distance between the given points P_1 and P_2.
   (a) $P_1(1, 1, 1)$, $P_2(3, 3, 0)$
   (b) $P_1(1, 4, 5)$, $P_2(4, -2, 7)$

6. Find the center and radius of the sphere $(x+2)^2 + y^2 + (z-2)^2 = 8$.

7. Find the equation of the sphere with center the point $(1, 2, 3)$ and radius $\sqrt{14}$.

8. Find the center and radius of the following spheres.
   (a) $x^2 + y^2 + z^2 + 4x - 4z = 0$.
   (b) $2x^2 + 2y^2 + 2z^2 + x + y + z = 9$.

9. Find a formula for the distance between the (arbitrary) point $P(x, y, z)$ to the x-axis, y-axis, and z-axis.

Solutions

1. Answer
   (a) the line parallel to the z axis and passing through the point $(2, 3, 15)$
   (b) the x-axis.
   (c) the circle of radius 2, centered at the origin and lying in the xy-plane
   (d) the circle of radius 2, centered at the origin and lying in the xz-plane
   (e) the circle of radius 1,centered at the origin and lying in the yz-plane
   (f) the circle of radius 4, centered at the origin and lying in the xy-plane

2. Answer
   (a) the closed (see problem 4) first quadrant of the xy-plane
   (b) the set of points strictly outside the sphere of radius 1, centered at the origin
   (c) the set of points on and above the xy-plane that are also inside or on the sphere of radius 1 that is centered at the origin.

3. Answer
   (a) $y = -1$
   (b) $y = -1$
   (c) $x = 0$, $(y-2)^2 + z^2 = 4$
   (d) $y = 3, \;\; z = -1$
   (e) $z = 3, \; x^2 + y^2 = 16$
   (f) Turn In problem

4. Answer
   (a) Interior: $(x-1)^2 + (y-1)^2 + (z-1)^2 < 1$.Exterior: $(x-1)^2 + (y-1)^2 + (z-1)^2 > 1$
   (b) Turn In problem

5. Find the distance between the given points P_1 and P_2.
   (a) 3
   (b) 7

6. Center: $(-2, 0, 2)$, radius $\sqrt{8}$.

7. $(x-1)^2 + (y-2)^2 + (z-3)^2 = 14$

8. Answer
   (a) Center: $(-2, 0, 2)$, radius: $\sqrt{8}$
   (b) Center: $\left(-\frac{1}{4}, -\frac{1}{4}, -\frac{1}{4}\right)$, radius: $\frac{5\sqrt{3}}{4}$

9. Answer
   (a) x-axis: $\sqrt{y^2 + z^2}$
   (b) y-axis: $\sqrt{x^2 + z^2}$
   (c) z-axis: $\sqrt{x^2 + y^2}$