

Double Integral Worksheet

Useful Properties of double integrals.

1. $\iint_D [f(x, y) + g(x, y)] dA = \iint_D f(x, y) dA + \iint_D g(x, y) dA$
2. $\iint_D c f(x, y) dA = c \iint_D f(x, y) dA$
3. If $f(x, y) \geq g(x, y)$ for all $(x, y) \in D$ then $\iint_D f(x, y) dA \geq \iint_D g(x, y) dA$
4. $\iint_D 1 dA = A(D)$
5. If $D = D_1 \cup D_2$ for non-overlapping “nice” regions, then $\iint_D f(x, y) dA = \iint_{D_1} f(x, y) dA + \iint_{D_2} f(x, y) dA$
6. If $m \leq f(x, y) \leq M$ for all $(x, y) \in D$ then $mA(D) \leq \iint_D f(x, y) dA \leq MA(D)$

Double Integrals over general regions in x, y coordinates

Sketch regions too

1. $\int_0^4 \int_0^{4-x} xy dy dx$
2. $\iint_D (x + y) dA$ where D is the triangle with vertices $(0, 0), (0, 2), (1, 2)$
3. $\iint_D 48xy dA$ where D is the region bounded by $y = x^3$ and $y = \sqrt{x}$

Reverse order of integration.

1. $\int_0^1 \int_x^{2x} e^{y-x} dy dx$
2. $\int_0^{2\sqrt{3}} \int_{y^2/6}^{\sqrt{16-y^2}} 1 dx dy$
3. $\int_0^7 \int_{x^2-6x}^x f(x, y) dy dx$
4. $\int_1^2 \int_x^{x^3} f(x, y) dy dx + \int_2^8 \int_x^8 f(x, y) dy dx$

Find Volume of solid

1. Tetrahedron in first octant bounded by coordinate planes and $z = 7 - 3x - 2y$.
2. Solid inside both the sphere $x^2 + y^2 + z^2 = 3$ and paraboloid $2z = x^2 + y^2$.

Double Integrals using polar coordinates

Direct Computations in polar coordinates

1. Compute $\int_0^{\pi/2} \int_1^3 r e^{-r^2} dr d\theta$
2. Find the area bounded by the cardioid $r = 1 + \sin \theta$.
3. Find the area bounded by one leaf of the rose $r = 4 \cos \theta$
4. Find area inside both $r = 1$ and $r = 2 \sin \theta$.

Convert from Cartesian (x, y) to polar coordinates before integrating

1. Find $\iint_D f(x, y) dA$ where D is the region bounded by the x -axis, the line $y = x$ and the circle $x^2 + y^2 = 1$.
2. Find the volume of the solid bounded by the paraboloid $z = 4 - x^2 - y^2$ and the xy -plane.
3. Find the volume inside the sphere $x^2 + y^2 + z^2 = 25$ and outside the cylinder $x^2 + y^2 = 9$.
4. Find the volume inside the sphere $x^2 + y^2 + z^2 = 25$ and outside the cylinder $(x - 1)^2 + y^2 = 1$.