Methods of Proof Handout 01

Direct Proof of $H \Longrightarrow C$

1. Start with the hypotheses of H .
2. Use nothing but allowable justifications.
3. Deduce C.

Direct Proof of $(H_1 \wedge \cdots \wedge H_n) \Longrightarrow C$

1. List all of the individual hypotheses $H_1, \cdots, H_n$ as given.
2. Use nothing but allowable justifications.
3. Deduce C.

Direct Proof of $H \Longrightarrow (C_1 \wedge \cdots \wedge C_n)$ **[Note this is equivalent to]** $((H \Longrightarrow C_1) \wedge \cdots \wedge (H \Longrightarrow C_n))$

1. Start with the hypotheses of H .
2. Use nothing but allowable justifications.
3. Deduce C_1.
4. Independently deduce each of the remaining C_i .

Use of the Contrapositive to prove $H \Longrightarrow C$

1. Present a direct proof of $\tilde{}C \Longrightarrow \tilde{}H$.
2. That is, Start with $\sim C$
3. Use nothing but logical steps
4. Deduce $\sim H$

Proof by Contradiction of $H \Longrightarrow C$ **(Not liked by Constructivists)**

1. Start with the hypothesis H .
2. Suppose the **RAA** hyptothesis $(\sim C)$
3. Use H , $(\sim C)$ and nothing but allowable justifications to deduce $(D \wedge (\sim D))$
4. Conclude $\sim (\sim C)$

How to deal with disjunctions

Disjoined Hypotheses $H_1 \vee \cdots \vee H_n \Longrightarrow C$

1. Do it by cases: Solve the n individual problems $H_1 \Longrightarrow C,\ H_2 \Longrightarrow C, \cdots, H_n \Longrightarrow C$

Disjoined Conclusions $H \Longrightarrow C_1 \vee \cdots \vee C_n$

1. Note: If any C_i can be deduced from H then the result is true.
2. Method of "hidden cases": Start with C and the negation of all but one C_i
3. Deduce this last C_i.

How to prove Universal statements $\forall x \ (p(x) \Longrightarrow q(x))$

1. Start with an **arbitrary** element x in the universal set X
2. Show that $p(x) \Longrightarrow q(x)$ using only the properties of x that make it an element of X,.

How to prove Existential Statements $\exists x \ p(x)$

1. Best approach is to **actually exhibit** an instance of x for which $p(x)$ is true.
2. If the above doesn't work, try a proof by contradiction.
   (a) Suppose for every x, $p(x)$ fails to be true and arrive at a contradiction.

Forward-Backward method for doing proofs

- Write out the hypotheses and the conclusions with space between
- Alternate between
  1. Logically deducing facts from the hypotheses
  2. Determining facts that imply the conclusions
  3. Join in the middle

Allowable Rules of Inference (deductions): these are all tautologies

1. **Modus Ponens** (mode that affirms) $((p \Longrightarrow q) \wedge p) \Longrightarrow q$
2. **Syllogism** $((p \Longrightarrow q) \wedge (q \Longrightarrow r)) \Longrightarrow (p \Longrightarrow r)$
3. **Modus Tollens** (mode that denies) $((p \Longrightarrow q) \wedge (\sim q)) \Longrightarrow (\sim p)$
4. **Contradiction** $((p \wedge (\sim q)) \Longrightarrow (r \wedge (\sim r))) \Longrightarrow q$
5. Tautology affirming using the **contrapositive** is valid: $(p \Longrightarrow q) \Longleftrightarrow ((\sim q) \Longrightarrow (\sim p))$

Terminology

- $((\sim q) \Longrightarrow (\sim p))$ is called the **contrapositive** of $(p \Longrightarrow q)$
- $(q \Longrightarrow p)$ is called the **converse** of $(p \Longrightarrow q)$
- $(\tilde{\ }p \Longrightarrow \tilde{\ }q)$ is called the **obverse** of $(p \Longrightarrow q)$