Mathematics 434 — Problem Sheet 5

Due February 28

Name

Directions: Be sure to include in-line citations, including page numbers if appropriate, every time you use the results of discussion, a text, notes, or technology. **Only write on one side of each page.**
"Experience is what enables you to recognize a mistake when you make it again." (Earl Wilson)

Problems

1. Do **both** of the following.

   (a) Prove all of these basic relations for rings.
   Let a, b, c be elements in a ring R. Then

   i. $a0 = 0a = 0$.
   ii. $a(-b) = (-a)b = -(ab)$
   iii. $(-a)(-b) = ab$
   iv. $a(b-c) = ab - ac$ and $(b-c)a = ba - ca$
   Furthermore, if R has a multiplicative identity 1, then
   v. $(-1)a = -a$
   vi. $(-1)(-1) = 1$

   (b) Given a ring R, the set of formal power series $p(t) = a_0 + a_1 t + a_2 t^2 + \cdots +$ ('formal' means there is no requirement of convergence) is a ring. (Denoted $R[[t]]$.) Show that $R[[t]]$ is a ring and prove that a formal power series $p(t)$ is invertible if and only if a_0 is a unit of R.

2. Let Q denote the rational numbers (you may use, without proof, the fact that Q is a field), $Q[\alpha]$ the smallest subring of C (the complex numbers) containing α, and $Q[\alpha, \beta]$ the smallest subring of C containing both α and β. Let $\alpha = \sqrt{2}$, $\beta = \sqrt{3}$ and $\gamma = \alpha + \beta$. Prove that $Q[\alpha, \beta] = Q[\gamma]$.

3. Using Peano's Axioms, prove the distributive law and the cancellation law of addition for the natural numbers. You may assume commutativity and associativity have already been proven.