Figure 1:

Spring 2010 **Final Exam** **Math** 300

May 12

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Name

Directions: Only write on one side of each page.

"In mathematics you don't understand things. You just get used to them." — John von Neumann
"Mystical explanations are considered deep. The truth is that they are not even superficial." — Friedrich Nietzsche

Extra Credit Make a list of statements that hold in real Euclidean planes but do not hold in real hyperbolic planes. One point (up to 6) for each correct statement that is not in the list of ten in Exercise 1 of Chapter 6 in the textbook. One point off (down to 0) for each incorrect statement.

You Must Do This Problem

Essay (20 points) State Meta Mathematical Theorem 1 and explain in detail why the material we covered from Chapter 7 proves this theorem.

Do any (5) of the following

1. (20 points) Explain why any statement T in the language of geometry that is a theorem in Euclidean geometry and whose negation is a theorem in hyperbolic geometry is equivalent to Hilbert's parallel postulate. [Hint: Let N denote the axioms of neutral geometry, H Hilbert's parallel postulate and T the given statement. Then part of what we know is $(N \wedge H) \Longrightarrow T$. You are asked to show: given N then $T \Longleftrightarrow H$.

2. (20 points) In the Poincaré disk model of hyperbolic geometry, suppose α is a Poincaré-line and P is an ordinary point not lying on α as in the figure. Explain how to construct the Poincaré-line that passes through P and is Poincaré-perpendicular to α.

3. (20 points) Using any result through Meta Mathematical Theorem 1, prove the Corollary to Meta Mathematical Theorem 1.

Figure 2:

Figure 3:

If Euclidean geometry is consistent then Hilbert's parallel property is independent of the axioms of neutral geometry.

4. (20 points) Prove directly that Incidence Axiom 3 holds in the Poincaré Disk interpretation. More specifically, translate Incidence Axiom 3 into a statement in Euclidean geometry using the Poincaré Disk interpretation and then prove that Euclidean statement.

5. (5, 15 points) In hyperbolic geometry, let l be a line and P a point not on l. Drop the perpendicular to l from P and call the foot Q.

   (a) State carefully what it means to say $\overrightarrow{PX}$ is a limiting parallel ray to l from P.

   (b) Prove that if Y is a point on the opposite side of line $\overleftrightarrow{PQ}$ from X for which $\measuredangle QPY \cong \measuredangle QPX$, then any ray between ray $\overrightarrow{PQ}$ and ray $\overrightarrow{PY}$ meets l. [You may use the fact that ray $\overrightarrow{PY}$ does not meet l without proving it.]

6. (20 points) Suppose γ, δ, and σ are circles in the Euclidean plane as shown in the figure on the blackboard. Suppose further that γ is orthogonal to both δ and σ and that O is exterior to all three circles and line $\overleftrightarrow{OT}$ is tangent to δ at T, line $\overleftrightarrow{OS}$ is tangent to γ at S and line $\overleftrightarrow{OR}$ is tangent to σ at S. Use the definition of the power of a point with respect to a circle and Lemma 7.1 to prove that $\overline{OT} = \overline{OS} = \overline{OR}$. Conclude that the circle ε with center O and radius $\overline{OT}$ is orthogonal to each of γ, δ,and σ.

7. (20 points) Using any results through Chapter 7, prove that in the Klein Disk model of hyperbolic geometry, if Klein line m is perpendicular to Klein line l then Klein line l is also perpendicular to Klein line m. More precisely, prove that if the extension of m passes through the pole of l, $P(l)$, then the extension of l passes through the pole of m, $P(m)$. [Hint: Use the circles α, β that are

orthogonal to γ_K and are centered at $P(l)$ and $P(m)$ respectively, the definition of "inverse point with respect to circles, and the power of $P(l)$ and $P(m)$ with respect to γ_k.]