Figure 1:

Math 300 Spring 2005

Final Exam

May 11, 2005

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Name

Directions: Be sure to include in-line citations, including page numbers if appropriate, every time you use a text or notes or technology. Include any figures used in solving a problem. **Only write on one side of each page.**

"Mystical explanations are considered deep. The truth is that they are not even superficial." — Friedrich Nietzsche

Do any five (5) of the following.

1. Prove the Corollary to Meta Mathematical Theorem 1.

   If Euclidean geometry is consistent then Hilbert's parallel property is independent of the axioms of neutral geometry.

2. Prove directly that Incidence Axiom 1 holds in the Poincaré Half-plane interpretation for any two Half-plane points that do not lie on a vertical Euclidean line. Specifically, describe the center and radius of the appropriate half-circle and explain why that half-circle satisfies the required properties.

3. Let l be a line in the Poincaré Half-plane model of hyperbolic geometry that is a vertical open ray (see the figure on the board). Let P be a point in the half-plane that is not incident with l. Carefully describe both limiting parallel rays to l from P and explain why they satisfy the definitition of limiting parallel rays.

4. Let δ be a line in the Poincaré Upper Half-Plane model of hyperbolic geometry and P a point not on δ as shown in the figure below. Describe (don't just sketch) how to find the Poincaré line α that passes through P and is orthogonal to δ.Specifically, describe the center and radius of the appropriate half-circle and explain why that half-circle satisfies the required properties.

Figure 2:

Figure 3:

5. In the Poincaré disk model of hyperbolic geometry, suppose α is a Poincaré-line and P is an ordinary point not lying on α as in the figure. Show how to construct the Poincaré-line that passes through P and is Poincaré-perpendicular to α.

6. Do **one** (1) of the following.

   (a) Prove that in the Klein Disk model of hyperbolic geometry, if Klein line m is perpendicular to Klein line l then Klein line l is also perpendicular to Klein line m. More precisely, prove that if the extension of m passes through the pole of l, $P(l)$, then the extension of l passes through the pole of m, $P(m)$. [Hint: Use the circles α, β that are orthogonal to γ_K and are centered at $P(l)$ and $P(m)$ respectively, the definition of "inverse point with respect to γ_K, and the power of $P(l)$ and $P(m)$ with respect to γ_k.]

   (b) Let γ be a circle used to specify both the Klein and Poincaré disk models of hyperbolic geometry ($\gamma_K = \gamma_P$). Use the fact from the previous problem that if Klein line m is perpendicular to Klein line l then l is also Klein perpendicular to m to prove that the Poincaré lines l_p and $m_p$that have the same ideal points as l and m, respectively are Poincaré perpendicular. [Hint: Use the circles that are orthogonal to γ centered at the poles of l and m, the definition of inverse points, and the power of $P(l)$ and $P(m)$.]

7. Make sure all of your sketches are carefully drawn. Let γ be a circle with center O and radius OR.

   (a) For the line l in the figure, draw (carefully sketch) the set of points l' that are the inverses with respect to γ of the points of l.

Figure 4:

Figure 5:

(b) For the line l in the figure, draw (sketch) the set of points l' that are the inverses with respect to γ of the points of l.

(c) For the line l in the figure, draw (sketch) the set of points l' that are the inverses with respect to γ of those points of l other than O.

(d) For the circle δ in the figure, draw (sketch) the set of points δ' that are the inverses with respect to γ of the points of δ.

(e) For the circle δ in the figure, draw (sketch) the set of points δ' that are the inverses with respect to γ of the points of δ. Note that δ is **not** orthogonal to γ.

(f) For the circle δ in the figure, draw (sketch) the set of points δ' that are the inverses with respect to γ of the points of δ. Note that δ **is** orthogonal to γ.

Figure 6:

Figure 7:

Figure 8:

Figure 9: