MATH 300 Spring 2005 Exam 3

April 19, 2005

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Name

Directions: Only write on one side of each page.

I. Do any (5) of the following

1. Do whichever of the following has your name. Note that this is an **if and only if** problem.

   (a) (Lindsey): Using any previous material, prove Proposition 4.8 which states *Hilbert's Parallel Postulate* $\Longleftrightarrow$ *Statement* $S.8$.

   Here statement $S.8$ is: (The converse to Theorem 4.1) If two parallel lines l, m are cut by a transversal t, then the alternate interior angles formed are congruent.

   (a) (Luke, Noa, Amy, Letani) Using any previous material, prove Proposition 4.7 which states *Hilbert's Parallel Postulate* $\Longleftrightarrow$ *Statement* $S.7$.

   Here statement $S.7$ is: If a line intersects one of two parallel lines, then it also intersects the other.

2. Guaranteed problem: (Exercise 4 of Chapter 6.)

   Using any material through Chapter 6 as well as any exercises before number 4 of Chapter 6, prove the following.

   Let l and l' be parallel lines with common perpendicular MM'. Let A and B be any points of l such that $M * A * B$, and drop perpendiculars AA' and BB' to l'. Prove that $AA' < BB'$.

3. In the figure on the board the pairs of angles ($\measuredangle A'B'B''$, $\measuredangle ABB''$) and ($\measuredangle C'B'B''$, $\measuredangle CBB''$) are called pairs of **corresponding angles** cut off on l and l' by transversal t. Prove that corresponding angles are congruent if and only if alternate interior angles are congruent.

4. Using any material through Chapter 5, prove that in neutral geometry there exists a triangle that is not isosceles.

5. Using any material through Chapter 6, prove the following.

   Let Neu denote the axioms of neutral geometry, Hil Hilbert's parallel axiom, and Hyp the hyperbolic parallel axiom. Show that any statement S in the language of neutral geometry that is a theorem in Euclidean geometry ($Neu + Hil \Longrightarrow S$) and whose negation is a theorem in hyperbolic geometry ($Neu + Hyp \Longrightarrow \tilde{}S$) is equivalent (in neutral geometry) to the parallel postulate. That is, given Neu, $S \Longleftrightarrow Hil$. [This is a slick way to find statements that are equivalent to the parallel postulate.]

6. In Theorem 4.1 it was proved in neutral geometry that if the alternate interior angles formed by a transversal to two lines are congruent, then the lines are parallel. Strengthen this result in hyperbolic geometry by proving that the lines have a common perpendicular. [Hint: Remember that in hyperbolic geometry lines can be parallel without having a common perpendicular so there really is something to prove here. To get you started, let M be the midpoint of the transversal segment PQ.]