Figure 1:

Math 213 **Spring 2005**

Final Exam

May 13, 2005

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Name

Directions: Be sure to include in-line citations, including page numbers if appropriate, every time you use a text or notes or technology. Include any figures used in solving a problem. **Only write on one side of each page.**

"Mystical explanations are considered deep. The truth is that they are not even superficial." — Friedrich Nietzsche

Do any five (5) of the following.

1. Essay question. State Meta Mathematical Theorem 1 and carefully explain the methodology we followed in our proof. You do not have to provide proofs for the verification of any of the axioms. As a **part** of your essay you should include a description of how we ended up with an interpretation that satisfied all of the axioms.

2. Prove the Corollary to Meta Mathematical Theorem 1.

   If Euclidean geometry is consistent then Hilbert's parallel property is independent of the axioms of neutral geometry.

3. Prove directly that Incidence Axiom 1 holds in the Poincaré Disk interpretation for any two points that do not lie on a diameter.

4. Let l be a line in the Poincaré Disk model of hyperbolic geometry that is not a diameter of γ_P.Let P be a point interior to γ_P that is not incident with l. Construct one of the limiting parallel rays to l from P and explain why it satisfies the definition of limiting parallel ray. Specifically, describe the center and radius of the appropriate circle δ and explain why that circle satisfies all of the necessary properties.

Figure 2:

Figure 3:

5. In the Poincaré disk model of hyperbolic geometry, suppose α is a Poincaré-line and P is an ordinary point not lying on α as in the figure. Show how to construct the Poincaré-line that passes through P and is Poincaré-perpendicular to α.

6. Prove that in the Klein Disk model of hyperbolic geometry, if Klein line m is perpendicular to Klein line l then Klein line l is also perpendicular to Klein line m. More precisely, prove that if the extension of m passes through the pole of l, $P(l)$, then the extension of l passes through the pole of m, $P(m)$. [Hint: Use the circles α, β that are orthogonal to γ_K and are centered at $P(l)$ and $P(m)$ respectively, the definition of "inverse point with respect to γ_K, and the power of $P(l)$ and $P(m)$ with respect to γ_k.]'

7. Be sure to draw all sketches in this problem carefully. Let γ be a circle with center O and radius OR.

   (a) For the line l in the figure, draw (carefully sketch) the set of points l' that are the inverses with respect to γ of the points of l.

Figure 4:

Figure 5:

(b) For the line l in the figure, draw (sketch) the set of points l' that are the inverses with respect to γ of the points of l.

(c) For the line l in the figure, draw (sketch) the set of points l' that are the inverses with respect to γ of those points of l other than O.

(d) For the circle δ in the figure, draw (sketch) the set of points δ' that are the inverses with respect to γ of the points of δ.

(e) For the circle δ in the figure, draw (sketch) the set of points δ' that are the inverses with respect to γ of the points of δ. Note that δ is **not** orthogonal to γ.

(f) For the circle δ in the figure, draw (sketch) the set of points δ' that are the inverses with respect to γ of the points of δ. Note that δ **is** orthogonal to γ.

Figure 6:

Figure 7:

Figure 8:

Figure 9: