Figure 1:

Spring 2010 **Exam 1** **Honors** 213

February 18

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Name

Directions: Only write on one side of each page.
From XKCD Webcomic.

Extra Credit

- (2 points): What is the negation of the statement "For every line l and every line m not equal to l, l and m are incident with exactly the same number of points"? You may use words, formal logical symbols, or a mixture of both.

Do any (5) of the following

1. (20 points) Give a detailed explanation of how and why we can use models to show that a statement S is independent of the axioms of an axiomatic system.

2. (10, 10 points) Given the following statement S: "For every line l and every line m not equal to l, l and m are incident with exactly the same number of points".

   (a) Present a model of Incidence geometry that shows it is impossible, using the axioms of incidence geometry, to prove statement S.

   (b) Present a model of Incidence geometry that shows it is impossible, using the axioms of incidence geometry, to prove the negation of statement S.

3. (20 points) Using any results through the corollary to Betweenness Axiom 4, prove the Same Side Lemma: Given $A * B * C$ and l and line other than line $\overleftrightarrow{AB}$ meeting line $\overleftrightarrow{AB}$ at point A. Then B and C are on the same side of line l.

4. (8, 8, 4 points) Show that it is possible for two four-point models of Incidence geometry to **not** be isomorphic by:

(a) **Carefully** stating what are the points, lines and incidence of both interpretations.

(b) Briefly illustrating why each is a model of Incidence geometry.

(c) Explaining how you know they are not isomorphic.

5. (20 points) Using any results from Incidence geometry, prove the following. In a finite affine plane in which every line has exactly 10 points then there cannot be more than 10 lines incident with any point. [Hint: start with an arbitrary point P and Proposition 2.4 and recall that an affine plane is a model of incidence geometry in which the Euclidean parallel property holds.]

6. (20 points) Using any previous results, give a formal proof of Proposition 2.1: If l and m are distinct lines that are not parallel then l and m have a unique point in common.

7. $(5, 15$ points) Proposition 2.6 says: For every point P there are at least two distinct points neither of which is P.

   (a) Restate this proposition in "If (hypothesis), then (conclusion)" form.

   (b) Using any previous results, give a formal proof of this proposition. [Be careful, there is nothing in the statement of the proposition that implies the point P is one of the points guaranteed by Incidence Axiom 3.]