Projective Plane Worksheet

Dual Planes

Suppose **P** is a projective plane.
We build a new interpretation **P′** of the undefined terms (point, line and incident) by swapping every occurrence of 'point' with 'line' and every occurrence of 'line' with 'point'. We retain the meaning of incidence.
Thus, the lines of **P′** are the points of **P** and the points of **P′** are the lines of **P**.

Proposition 1 *The interpretation* **P′** *is also a projective plane.*

We prove this by translating what we wish to know about **P′** into statements we know to be true in **P**.

P		**P′**
Holds in **P**	Translates to	Want to Hold in **P′**
	$\rightleftharpoons$	IA.1(1): $\forall$ distinct pts $l\,,m$ $\exists$ line P incident with both $l\,,m$
	$\rightleftharpoons$	IA.1(2): $\forall$ distinct pts $l\,,m$ $\exists$ only one P incident with both $l\,,m$
	$\rightleftharpoons$	IA.2: $\forall$ line P $\exists$ two distinct pts l , m both incident with P
	$\rightleftharpoons$	IA.3: $\exists$ three distinct pts with no line incident with all three
	$\rightleftharpoons$	Prop.2.1: $\forall$ distinct non-parallel lines P, Q, $\exists$ only one pt l inc with both
I.A. 3	$\rightleftharpoons$	Prop.2.2: $\exists$ three distinct lines with no point incident with all three
	$\rightleftharpoons$	Prop. 2.3: $\forall$ lines P $\exists$ at least one point l not incident with it.
	$\rightleftharpoons$	Prop.2.4: $\forall$ point l $\exists$ at least one line P not incident with it.
	$\rightleftharpoons$	Prop.2.5: $\forall$ point l $\exists$ at least two lines incident with l
	$\rightleftharpoons$	Elliptic Parallel Prop.: $\forall$ distinct lines P, Q $\exists$ point l incident with both.
	$\rightleftharpoons$	Strong IA.2: $\forall$ line P $\exists$ at least three distinct pts $l\,, m, n$ incident with P