

April 16

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Name

**Directions:** Only write on one side of each page.

**Do any (5) of the following**

1. Using any previous results, prove Proposition 4.1 (SAA) in neutral geometry. Specifically, Given  $AC \cong DF$ ,  $\angle A \cong \angle D$ , and  $\angle B \cong \angle E$ . Then  $\triangle ABC \cong \triangle DEF$ .
2. Using any previous results, prove the following half of Proposition 4.10.  
(If  $k \parallel l$ ,  $m \perp k$ , and  $n \perp l$ , then either  $m = n$  or  $m \parallel n$ .) implies Hilbert's Euclidean parallel postulate.
3. Prove
  - (a) Every acute angle has a complementary angle.
  - (b) If the complements of two acute angles are congruent then the acute angles are congruent.
4. A scalene triangle is defined to be any triangle that is not isosceles. Using any results through the end of Chapter 4, prove that in any Hilbert plane there is a triangle that is scalene.
5. Here is a statement  $S_p$ : Given lines  $l, m, n$ . If  $l \parallel m$  and  $m \parallel n$ , then  $l \parallel n$ .  
Using any results through Chapter 4, prove  $S_p$  holds if and only if Hilbert's Euclidean parallel postulate holds.
6. Using any result through the Chapter 4, prove the following.  
If  $\square ABCD$  is a convex quadrilateral and  $l$  is any line other than  $\overleftrightarrow{AB}$  intersecting segment  $AB$  in a point between  $A$  and  $B$ , then  $l$  also intersects at least one of  $BC, CD, AD$ .