Smith **Math 290** **Spring** 2010

Proof R-1

Accepted **Not Accepted**

I affirm this work abides by the university's Academic Honesty Policy.

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Print Name, then Sign

- First due date **Thursday, December** 4.
- Turn in your work on a separate sheet of paper with this page stapled in front.
- Do not include scratch work in your submission.
- There is to be **no collaboration** on any aspect of developing and presenting your proof. Your only resources are: you, the course textbook, me, and pertinent discussions that occur **during class.**
- Follow the Writing Guidelines of the Grading Rubric.
  (http://math.ups.edu/~bryans/Current/Fall_2008/290inf_Fall2008.html#tth_sEc5.1)
- Retry: Only use material from the relevant section or earlier.
- Retry: Start over using a new sheet of paper.
- Retry: Restaple with new attempts first and this page on top.

The perplexity of life arises from there being too many interesting things in it for us to be interested properly in any of them. – G. K. Chesterton, 1909

R-1 (You may use material up through Section MR)
Let U, V be vector spaces and $T : U \longrightarrow V$ a linear transformation.

1. Let $\{\vec{c}_1, \vec{c}_2, \cdots, \vec{c}_k\}$ be a basis for $R(T)$,the range of T and for each i, $1 \leq i \leq k$ let $\vec{b}_i$ be a vector in $T^{-1}(\vec{c}_i)$, the preimage of $\vec{c}_i$.

   Prove that the set $\left\{\vec{b}_1, \vec{b}_2, \cdots, \vec{b}_k\right\}$ is linearly independent in U.

2. Consider the specific linear transfomation $T : \mathbf{C}^3 \longrightarrow \mathbf{C}^3$ given by $T(\vec{x}) = A\vec{x}$ where $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$

   (a) Use the 'L' matrix of the extended–echelon form for matrix A to find a basis $Q = \{\vec{c}_1, \cdots, \vec{c}_k\}$ for the range of T.

   (b) Extend this set to a basis C of the codomain $\mathbf{C}^3$ by "adding" appropriate additional vector(s).

   (c) For each vector $\vec{c}_i$ in S, compute a vector $\vec{b}_i$ in the preimage $T^{-1}(\vec{c}_i)$ and collect these vectors into a set P.

   (d) Compute a basis for the kernel of T, $K(T)$ and let B be the union of this set and the set P.

   (e) Cite a theorem from the text whose proof shows that B is a basis for the domain of T.

   (f) Compute the matrix representation $M^T_{B,C}$.
