

Proof VS-2

Accepted

Not Accepted

I affirm this work abides by the university's Academic Honesty Policy.

Print Name, then Sign

- First due date **Thursday, April 8**.
- Turn in your work on a separate sheet of paper with this page stapled in front.
- Do not include scratch work in your submission.
- There is to be **no collaboration** on any aspect of developing and presenting your proof. Your only resources are: you, the course textbook, me, and pertinent discussions that occur **during class**.
- Follow the Writing Guidelines of the Grading Rubric in the course information sheet.
- Retry: Only use material from the relevant section of the text or earlier.
- Retry: Start over using a new sheet of paper.
- Retry: Re-staple with new attempts first and this page on top.

"Experience is what enables you to recognize a mistake when you make it again." (Earl Wilson)

 VS-2 (Section PD)

Suppose $S = \{\vec{v}_1, \vec{v}_2, \vec{v}_3, \dots, \vec{v}_{p-1}, \vec{v}_p, \vec{v}_{p+1} \dots \vec{v}_m\}$ is an orthonormal basis for \mathbf{C}^m and let $V = \langle \{\vec{v}_1, \vec{v}_2, \vec{v}_3, \dots, \vec{v}_{p-1}, \vec{v}_p\} \rangle$ be the subspace of \mathbf{C}^m spanned by the first p vectors in S and $W = \langle \{\vec{v}_{p+1} \dots \vec{v}_m\} \rangle$ be the subspace of \mathbf{C}^m spanned by the last $m - p$ vectors in S .

1. Quote the theorem from our textbook that tells us that $\mathbf{C}^m = V \oplus W$.
2. Prove that if $\vec{w} \in W$, then \vec{w} is orthogonal to every vector in V .
3. Prove that if \vec{x} is orthogonal to every vector in V , then $\vec{x} \in W$.

Notes

- Because W satisfies the two properties (2 and 3) above, it is called the **orthogonal complement** of V in \mathbf{C}^m and is usually written V^\perp .
- Professor Beezer has proved that
 1. Every subspace, V , of \mathbf{C}^m has a basis
 2. That basis can be extended to a basis of \mathbf{C}^m and
 3. The Gram-Schmidt procedure can transform any basis into an orthonormal basis.

Your work along with these details is enough to prove the theorem

Theorem 1 *If V is a subspace of \mathbf{C}^m then $\mathbf{C}^m = V \oplus V^\perp$*