

# Honors 213, Foundations of Geometry

## Goals

**Non-Euclidean Geometry** We will explore the 2000 year history and development of one of the most perplexing, and hence interesting, ideas to arise in western civilization. Our approach will follow that of our author (Greenberg) who uses historical vignettes as motivation and the axiomatic method as the primary tool for understanding the basics of logic, proof, Euclidean, and non-Euclidean Geometry.

**Reading/Writing** It is important that you read the text. In fact, developing the ability to read technical material with understanding is one of the primary goals of this course. Another goal is to fine-tune the ability to present written arguments clearly and gracefully. You will find that mathematical writing's focus on clarity gives a canonical framework for all of your expository writing.

**Proof** Most of this course, either directly or indirectly, is pointed at the related issues of proof and communication. In particular, you will learn what it means when a mathematician claims to have “proved” a fact and, through the assigned paper, you will have the opportunity to explore other notions of communication. Our tools for this study of mathematical proof are exactly the same as those used by our primary author (Greenberg) in his presentation of non-Euclidean geometry: elementary formal logic and the axiomatic method. We will use our other texts (Euclid, Bolyai, Lobachevskii) to provide additional historical context to our studies.

## Course Information

**TEXTS** *Euclidean and Non-Euclidean Geometries* [4], Fourth Edition, Marvin Jay Greenberg, W.H. Freeman and Company, 2008.

The Elements of Euclid, Robert Simson, 1806. The Open Library [9]

Euclid's Elements of Geometry, John Keill, MDCCXXIII. Courtesy of the Internet Archive [7]

The Science Absolute of Space, John Bolyai, Translated by George Bruce Halsted, The Neomon, 1896. Courtesy of the Internet Archive. Also available at Amazon.com for \$20 [5]

Geometrical Researches on the Theory of Parallels, N. Lobachevskii, Open Court, Translated by George Bruce Halsted, 1914. Courtesy of the Internet Archive. (Also available at Amazon.com for \$15.00) [8]

## Logistics

<b>Prof. Bryan Smith</b>	TH 390D	879-3562	bryans[at]ups.edu		
Hon 213A	TH 387	M,T,Th,F	2:00 - 2:50 P.M.		
<b>Office Hours</b>		Tue, Thu	9:30-10:00 AM	1:30-2:00 PM	3:00-3:30 PM
		Wed	3:00-4:30 PM		
		Others	By Appointment		

I am also available at other times – say, the half hour before the Tue, Thu, Office Hour as well as by appointment.

**Day to Day Structure** The class days will typically be devoted to interactive lectures on material in our textbooks. Expect at least one “Brainstorming” day for each homework assignment and occasional days working at the blackboard.

**EXAMINATIONS** There will be three, 100 point, one hour, in-class examinations. Make-up examinations are at my discretion and their existence has the necessary (but not sufficient) condition that you make arrangements prior to the exam. Sufficient interest from the class can change examination dates or move the exams to a 2-hour, evening format. Examinations will typically consist of three questions from the homework and two questions you have not seen before. When taking the examinations you will be allowed to use your copy of my handout of definitions and theorems as a personalized resource. The examinations are tentatively scheduled for the following days:

Examination One      Thursday February 18  
Examination Two      Thursday March 25  
Examination Three    Thursday April 22

**Study Session** If there is enough interest we can schedule an evening study session for the Wednesday before an examination.

**FINAL EXAMINATION: Monday May 10, 4:00 P.M.** The Final will actually be a fourth examination on the material covered since the third in-class examination.

## HOMEWORK

There will be homework assignments every week and problems will be graded using the rubric attached at the end of these notes. Think of these take-home assignments as weekly papers in which you first analyze and solve a problem and then completely explain that analysis and solution. At the very least you should type your papers using complete sentences, in the first person plural, with accurate punctuation, for an audience consisting of students not in this class but with an equivalent background, and in a clear, easy to follow expository style.

You are not to work with anyone when doing these assignments. However,

- you may use (with citation) any idea verbalized during a “brainstorming” session.
- you may use (with citation) any idea you obtain in discussion with me.
- you may collaborate with other students if I explicitly allow you to do so.

As long as you cite it, feel free to use (or not) any technology that you like (e.g., Sage, CABRI, Geometers Sketchpad, calculators, *Mathematica*, MATLAB, etc.).

## Paper

There will be one paper assigned around midterm.

The primary goal of this course is for you to learn the basics of ‘mathematical reasoning’ or ‘mathematical ways of knowing’. This is addressed in the course itself by giving you extensive practice at the mathematical use of the related concepts of proof, communication, meaning, and truth. To provide contrast with or context for this mathematical methodology, your paper will examine how some other discipline (your major, minor or some other field of interest) approaches these same fundamental concepts. You can find examples of such papers at

The Journal of Undergraduate Mathematics at Puget Sound [10]

These ideas are at the heart of much philosophical debate and have been the subject of many large books. But you will only have 2-4 weeks for your paper so keep your topic tightly focussed. Remember, the most important goal of the paper is for you to *begin* to explore such philosophical fundamentals as proof, communication, and truth.

## Paper Logistics

**Before February 26:** Meet with me to discuss your list of possible topics for your paper.

**Due March 9:** Last day to have a topic paragraph **accepted**.

**Due April 9:** Turn in 3 copies of a draft of the paper (please print on both sides to save paper). I will distribute 2 of those copies to your referees. (This means each of you will referee two papers.) The referees will comment (in written form) on the accuracy, clarity of exposition and appropriateness of these papers for the *Journal of Undergraduate Mathematics at Puget Sound* as outlined in the *Journal Guidelines for Authors*. [10]

**Due April 16** Referees give their reports to authors.

**Due April 23:** Turn in the final version of the paper in both paper and electronic (.PDF, .PS or .DOC) form along with all referee comments. If the paper receives a passing grade, it will be published in the journal.

The author will receive a grade for the paper itself and the referees will receive grades for the quality of their comments.

Students who are majoring in mathematics or science should not use a standard word processor for writing their paper. Instead, they should use the L<sup>A</sup>T<sub>E</sub>X typesetting package. For those who are interested, I will be happy to give a brief workshop on how to install and use this package.

## Course Information Updates

If you wish, I will post (and update throughout the semester) a grade report on your current class standing on my university web page. You should keep track of your grades on the various assignments and check them against these reports. If there are any discrepancies they should be dealt with immediately.

To have your information posted you need to print your name, the class (HON 213), and a code on a sheet of paper. Then sign the paper and physically hand it to me. The code must be a sequence of up to 23 symbols I can type on a keyboard.

## TOTAL POINTS

Homework	54%
Paper	10%
Referee Reports	4%
Examinations	24%
Final Examination	8%

## First Assignment

**(Due Friday January 22)** Find my university web page

(<http://math.ups.edu/> → faculty → Bryan Smith)

and locate the *Journal of Undergraduate Mathematics at Puget Sound* “Guidelines for Authors” page. Then send an e-mail message to me at [bryans@ups.edu](mailto:bryans@ups.edu). This homework assignment will not be complete unless you receive an email from me acknowledging your email message.

# References

- [1] Bryan Smith's Homepage  
<http://math.ups.edu/~bryans/>
- [2] Course Webpage for Honors 213A  
[http://math.ups.edu/~bryans/Current/Spring\\_2009/213Index\\_Spring2009.html](http://math.ups.edu/~bryans/Current/Spring_2009/213Index_Spring2009.html)
- [3] Department Syllabus for Honors 213  
[http://www.math.ups.edu/~matthews/Syllabi/Honors\\_213\\_Syllabus.pdf](http://www.math.ups.edu/~matthews/Syllabi/Honors_213_Syllabus.pdf)
- [4] "Euclidean and Non-Euclidean Geometries" Fourth Edition, Marvin Jay Greenberg, W.H. Freeman and Company, 2008.
- [5] "The Science Absolute of Space", John Bolyai, Translated by George Bruce Halsted, 1896. Courtesy of Google (Search on Title and Bolyai) (Also available through Amazon.com)
- [6] "The First Six Books of the Elements of Euclid", John Casey (and Euclid), Project Gutenberg Online Book Catalog. <http://www.gutenberg.org/etext/21076>
- [7] "Euclid's Elements of Geometry", Dr John Keill, Half-Moon, MDCCXXIII. Courtesy of the Internet Archive <http://www.archive.org/details/euclidselements02keilgoog>
- [8] "Geometrical Researches on the Theory of Parallels", N Lobachevskii, Open Court, Translated by George Bruce Halsted, 1914. Courtesy of the Internet Archive (Also available at Amazon.com)
- [9] "The Elements of Euclid, viz The First Six Books Together with the Eleventh and Twelfth", Robert Simson, MD, J. Conrad & Co, 1806. Courtesy of the Open Library Project [http://openlibrary.org/b/OL23297112M/Elements\\_of\\_Euclid](http://openlibrary.org/b/OL23297112M/Elements_of_Euclid)
- [10] *The Journal of Undergraduate Mathematics at Puget Sound*  
<http://math.ups.edu/~bryans/Current/HTML/journalhome.html>  
<http://math.ups.edu/~bryans/Current/HTML/JournalGuidelines.html>
- [11] William Rapaport's "How to Study"  
<http://www.cse.buffalo.edu/~rapaport/howtostudy.html>

Points	Logic and Mathematics
5	Arguments are correct, complete and without extraneous or misleading material.
4	Arguments have only one of: a few minor errors, omissions or inappropriate material.
2	Arguments have at least two of: minor errors, omissions and inappropriate material.
0	Arguments are more seriously flawed.
Points	Use of Terminology and Notation
2	All technical terms, concepts and notation are used correctly.
1	There are minor problems with terminology and or concepts.
0	There are major problems with terminology or concepts.
Points	Written Presentation
3	Follows citation requirements and all other writing guidelines.
2	Follows almost all of the guidelines with only one or two minor lapses.
1	Has more than one or two minor lapses on following the guidelines.
0	Has a major lapse in following the guidelines.

## Writing Guidelines

It is best to think of these writing projects as officially assigned papers in which you completely explain and justify your analyses of the problems. Unless I say otherwise, there is to be no collaboration at all when you work these problems and write them up. Your sole outside resources are direct discussions with me or discussions that occur during class.

In addition I expect your papers to be

- Fully documented – specifically:
  1. Any idea obtained during in-class brainstorm sessions is cited in-line.
  2. All textbook results (theorems, propositions, and lemmas) are cited in-line and include the name of the result.
  3. Any use of technology is cited in-line.
- Written with a word processor. (I can show you how to install L<sup>A</sup>T<sub>E</sub>X, use Scientific Notebook (in the labs) or you can use Mathematica or Microsoft Word. Please check with me before using any other program.)
- Written using complete, accurately punctuated sentences.
- Presented in active voice, the first person plural and with a clear, easy-to-follow expository style.
- Targeted at an audience consisting of students not in this class but with an equivalent mathematical background – say those currently in another section of this course.