Turn In 4.1

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Vector Proof

We prove that if $\vec{u} \neq \vec{0}$ and both $\vec{u} \cdot \vec{v} = \vec{u} \cdot \vec{w}$ and $\vec{u} \times \vec{v} = \vec{u} \times \vec{w}$ then it must be the case that $\vec{v} = \vec{w}$.

Proof: We note first that the angle between two **non-zero** vectors is $\pi/2$ if and only if their dot product is zero and that the angle between two **non-zero** vectors is 0 or π if if and only if their cross product is zero. (See pages 630 and 637 of the textbook).

We also note (using the properties on page 631 and 637, that $\vec{u} \cdot \vec{v} = \vec{u} \cdot \vec{w}$ and $\vec{u} \times \vec{v} = \vec{u} \times \vec{w}$ imply

$$\begin{aligned} \vec{u} \cdot \vec{v} &= \vec{u} \cdot \vec{w} \\ \vec{u} \cdot \vec{v} - \vec{u} \cdot \vec{w} &= 0 \\ \vec{u} \cdot (\vec{v} - \vec{w}) &= 0 \end{aligned}$$

and

$$\begin{aligned} \vec{u} \times \vec{v} &= \vec{u} \times \vec{w} \\ \vec{u} \times \vec{v} - \vec{u} \times \vec{w} &= \vec{0} \\ \vec{u} \times (\vec{v} - \vec{w}) &= \vec{0} \end{aligned}$$

Putting this together we see that if $\vec{u}$ and $(\vec{v} - \vec{w})$ are two **non-zero** vectors, then theangle between them must simultaneously be $\pi/2$ and one of 0 or π. Since this can't happen we know that at least one of $\vec{u}$ and $(\vec{v} - \vec{w})$ must be the zero vector but our problem statement says that it isn't $\vec{u}$ so it must be that $\vec{v} - \vec{w} = \vec{0}$ which tells us that $\vec{v} = \vec{w}$.