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#48 p. 627

Find coordinates of point  $Q$  that divides the segment between  $P_1(x_1, y_1, z_1)$  and  $P_2(x_2, y_2, z_2)$  into two lengths ~~whose~~ whose ratio is  $p/q = r$ . (Special case where  $p=3$ ,  $q=5$ , and the distance between  $P_1$  and  $P_2$  is 16)

To find the coordinates of point  $Q$ , we can modify the formula for the midpoint of a segment to fit the given parameters of this problem. Page 625 of the textbook shows both this formula and its proof.

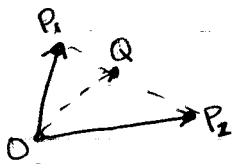


Figure 1

Figure 1 shows a diagram that is useful in using the given formula, showing  ~~$\vec{P_1}$~~  and  ~~$\vec{P_2}$~~  with vectors with the same base and end points at  $P_1$  and  $P_2$ , and another vector with tip at point  $Q$ . We start with the equation  $\vec{OQ} = \vec{OP_1} + \frac{3}{8}(\vec{P_1P_2})$ , because one segment of  $\vec{P_1P_2}$  is  $3k$  (where  $k$  is an unknown constant) and the other is  $5k$ , so the ratio of the segment  $\vec{P_1Q}$  to  $\vec{P_1P_2}$  is  $3/8$ . Following the book's proof, we simplify this equation to  $\vec{OQ} = \frac{3}{8}\vec{OP_2} + \frac{5}{8}\vec{OP_1}$ , which in turn is equal to  $\frac{3}{8}\langle x_2, y_2, z_2 \rangle + \frac{5}{8}\langle x_1, y_1, z_1 \rangle$ . Finally we combine these two vectors and get their endpoint  ~~$Q$~~   $Q(\frac{5}{8}x_1 + \frac{3}{8}x_2, \frac{5}{8}y_1 + \frac{3}{8}y_2, \frac{5}{8}z_1 + \frac{3}{8}z_2)$ . — excellent

#28 p.

If  $\vec{u} \cdot \vec{v}_1 = \vec{u} \cdot \vec{v}_2$ , can we then say that  $\vec{v}_1 = \vec{v}_2$ ?

To begin, we know we can not simply "cancel"  $\vec{u}$  (divide by  $\vec{u}$ ) on both sides of the given equation because we have no definition for division of vectors. } excellent.  
Expanding the first equation, ~~we get~~  $\vec{u} \cdot \vec{v}_1 = \vec{u} \cdot \vec{v}_2$ , we have  $u_1(v_{11}) + u_2(v_{12}) + u_3(v_{13}) = u_1(v_{21}) + u_2(v_{22}) + u_3(v_{23})$ .  
From here we can see that with nine variable values for each entry of the three vectors, there must be infinitely many vectors  $\vec{v}_1$  and  $\vec{v}_2$  that make this equation true, of which vectors  $\vec{v}_1 = \vec{v}_2$  are only the most obvious. For example, if we have  $\vec{u} = \langle 2, 3, 2 \rangle$ ,  $\vec{v}_1 = \langle 4, 1, 3 \rangle$ ,  $\vec{v}_2 = \langle 0, 3, 4 \rangle$ , ~~the~~ the dot product of  $\vec{u} \cdot \vec{v}_1$  equals 17, as does the dot product  $\vec{u} \cdot \vec{v}_2$ , even though  $\vec{v}_1$  is not equal to  $\vec{v}_2$ .

Good