Math 290   Spring 2007   Project 05

Due April 13

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Name

Be sure to re-read the WRITING GUIDELINES rubric, since it defines how your project will be graded. In particular, you may discuss this project with others but **you may not collaborate on the written exposition of the solution.**

"It is hard to know what you are talking about in mathematics, yet no one questions the validity of what you say. There is no other realm of discourse half so queer." – J. R. Newman

Another Dynamical System

Imagine that you are diabetic, and so you have to pay close attention to how your body metabolizes glucose. Let $g(t)$ represent the **excess glucose concentration** in your blood, usually measured in milligrams of glucose per 100 milliliters of blood. (Excess means that we measure how much the glucose concentration deviates from your fasting level, i.e., the level your system approaches after many hours of fasting.) A negative value of $g(t)$ indicates that the glucose concentration is below fasiting level at time t. Shortly after you eat a heavy mean, the function $g(t)$wil reach a peak, and then it will slowly return to 0. Certain hormones, notably insulin, help regulate glucose. Let $h(t)$ represent the **excess hormone concentration** in your blood. Researchers have developed mathematical models for the glucose regulatory system. Here is one such (slightly simplified) model (these formulas apply between meals; the system is disturbed during and right after a meal).

$$\begin{aligned} g(t+1) &= ag(t) - bh(t) \\ h(t+1) &= cg(t) + dh(t) \end{aligned}$$

where time t is measured in minutes, a and d are constants slightly less than 1 and b and c are small positive constants. For the purposes of this project, use the system where $a = 0.978$, $b = 0.006$, $c = 0.004$, and $d = 0.992$.

Questions

1. What does the negative coefficient in the model tell you about the effect of insulin on the glucose level of the body? What does the positive constant c tell you about the effect of glucose on the level of insulin in the body?

2. Describe this system in the form $\vec{x}(t+1) = A\vec{x}(t)$ using a matrix A and find eigenvalue, eigenvector pairs for this matrix. Select your eigenvectors so that they have integer entries.

3. After you have consumed a heavy meal, the concentrations in your blood are $g_0 = 100$ and $h_0 = 0$. Find closed formulas for $g(t)$ and $h(t)$. Sketch the endpoints of the vectors $\vec{x}(t) = \begin{bmatrix} g(t) \\ h(t) \end{bmatrix}$ for $t = 0, 10, 20, \cdots, 70$. Briefly describe in practical terms how this glucose-hormone system evolves over time.

4. For the special initial conditions in the previous question, determine how long it takes for the glucose concentration level to fall below fasting level. (This quantity is useful in diagnosing diabetes in that a period of more than four hours may indicate mild diabetes.

Citations

- Ackerman et al., "Blood glucose regulation and diabetes," Chapter 4 in *Concepts and Models of Biomathematics*, Marcel Dekker, 1969.

- Otto Bretscher, *Linear Algebra with Applications,* Prentice Hall 1997.