Math 290 **Spring 2007** **Project 01**

Due January 26

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Name

Be sure to re-read the WRITING GUIDELINES rubric, since it defines how your project will be graded. In particular, you may discuss this project with others but **you may not collaborate on the written exposition of the solution.**
"Mathematics is the language with which God has written the universe" -Galileo Galilei, physicist and astronomer (1564-1642)

Leontief Input-Output Models

The U.S. economist won the Nobel prize in economics for his work on the following question: What output should each of the industries in an economy produce to satisfy the total demand for all the products? His solution was to model the economy using a (large) system of linear equations to encode how each industry relied on each of the other industries. Here is a simple examle. Suppose an economy has exactly two industries: A and B. Assume that the consumer demand for their products is, respectively, $1,000$ and 780, in millions of dollars per year. In order to determine the outputs, a and b (in millions of dollars per year) the two industries should generate to satisfy the demand we also have to take into account the interindustry demand. For example, suppose industry A produces electricity and that industry B needs \$0.10 worth of electricity for each \$1.00 of output B produces. Suppose further that industry A needs \$0.20 worth of B's products for each \$1.00 of output A produces. If we focus on industry A we see that A must be able to satisfy both the consumer demand and B's demand so the output of A (which we denote by a) can be summarized by: $a = 1000 + 0.1b$. Similarly, the output of B is summarized by $b = 780 + 0.2a$. It is now straightforward to compute the outputs a and b by solving the linear system of equations

$$\begin{aligned} a - 0.1b &= 1000 \\ -0.2a + b &= 780 \end{aligned}$$

Of course, using Leontief's techniques to model the economy of a country (Japan actually did this) requires far more than two industries so we need to introduce notation that will allow us to consider thousands of industries.

Suppose we designate our distinct industries by $I_1, I_2, I_3, \cdots, I_n$ and their respective outputs by $x_1, x_2, x_3, \cdots, x_n$. For each i, $1 \le i \le n$, we use b_i to denote the consumer demand for industry I_i and we use a_{ij} to denote the demand that industry I_j puts on industry I_i for each \$1.00 of output that industry I_j produces. For example, $a_{3,2} = 0.5$ means that industry I_2 needs \$0.50 worth of products from industry I_3 for each \$1.00 of goods industry I_2 produces. Note that the numbers a_{ii} need not be zero since, for example, an industry producing electricity needs electricity to run its machinery.

1. Explain each of the following.

   (a) What is the meaning in economic terms of $x_1a_{i1} + x_2a_{i2} + x_3a_{i3} + \cdots + x_na_{in} + b_i$.?

   (b) What is the meaning in economic terms of the equation $x_1a_{i1}+x_2a_{i2}+x_3a_{i3}+\cdots+x_na_{in}+b_i = x_i$?

2. Showing your work, find the outputs x_1, x_2, x_3 required to satisfy the demand of this example economy taken from Leontief's book *Input-Output Economics,* Oxford University Press, 1966.

   There are three industries: I_1 is agriculture, I_2 is manufacturing, and I_3 is energy. Outputs and demands are measured in millions of Israeli pounds, the currency of Israel at that time. We are told the various demand numbers which I have organized below for ease of reading.

$$\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = \begin{bmatrix} 13.2 \\ 17.6 \\ 1.8 \end{bmatrix} \text{ and}$$

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = \begin{bmatrix} 0.293 & 0 & 0 \\ 0.014 & 0.207 & 0.017 \\ 0.044 & 0.01 & 0.216 \end{bmatrix}$$