Math 232 Spring 2006 Project 08

Due March 24

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Name

Be sure to re-read the WRITING GUIDELINES rubric, since it defines how your project will be graded. In particular, you may discuss this project with others but **you may not collaborate on the written exposition of the solution.**

"Anyone who cannot cope with mathematics is not fully human. At best he is a tolerable subhuman who has learned to wear shoes, bathe, and not make messes in the house." – Robert Heinlein in *Time Enough for Love.*

Do one (1) of the computation problems (but understand the other).

C.1. Suppose V is a subspace of $\mathbf{C}^n$ and consider the set, W, of vectors orthogonal to every vector in V. More specifically, $W = \{\overrightarrow{w} \in R^n \mid <\overrightarrow{w}, \overrightarrow{v}> = 0, \text{ for every } \overrightarrow{v} \in V\}$ [W is called the **orthogonal complement of** V and is sometimes denoted $V^{\perp}$].

(a) Show that $W = V^{\perp}$ is a subspace of $\mathbf{C}^n$.

(b) Let V be the subspace of $\mathbf{C}^4$ with basis $\left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} \right\}$. Find a basis for $W = V^{\perp}$. Be sure to show your set both spans W and is linearly independent.

C.2. **Definition**: A matrix $A \in M_{nn}$ is said to be **skew symmetric** if $A^t = -A$.

(a) Show that the set $W = \{A \in M_{nn} : A^t = -A\}$ of skew symmetric matrices in M_{nn} is a subspace of M_{nn}.

(b) Find a basis for the subspace of skew symmetric matrices in M_{33}. Be sure to show your set both spans W and is linearly independent.

Do this theoretical problem.

T.1. Let V be an arbitrary vector space. Show that the set of subspaces of V is closed under intersections. More specifically: prove that if U, W are subspaces of V then the set $U \cap W$ is also a subspace of V.

Nota Bene

1. The set of subspaces of a vector space need not be closed under unions. That is, if U, W are subspaces of a vector space V then $U \cup W$ does not need to be a subspace of V.

   For example, if $V = \mathbf{C}^2$, $U = <\begin{bmatrix} 1 \\ 0 \end{bmatrix}>$, and $W = <\begin{bmatrix} 0 \\ 1 \end{bmatrix}>$. Then $\vec{u} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \in U \subseteq U \cup W$ and $\vec{w} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \in W \subseteq U \cup W$ but $\vec{u} + \vec{w} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \notin U \cup W$ because if it were it would have to be in U or in W but it cannot be written as $\alpha \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and hence is not in U nor can it be written as $\beta \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ so it is not in W. This shows that $U \cup W$ does not satisfy the three properties necessary to be a subspace of V.

2. Occasionally, during the remainder of the semester, I will introduce examples where the only numbers we use are real numbers – not complex. In order to make this more rigorous, here is the pertinent definition.

   Definition: In the definition of vector space (Definition VS, page 303 of our text) scalar multiplication is defined using **complex** numbers as the scalars. If, instead, we require the scalars to be **real** numbers but keep every other part of the definition of vector space, then the set V with its addition and scalar multiplication (by real scalars) is called a **Real vector space**. Thus the only difference between the vector spaces we have been considering and a real vector space is that in the latter we restrict ourselves to only using real scalars. If you look at the proofs of the theorems thus far, you will see that the only properties of the complex numbers we have used (closed under addition and multiplication, associative, commutative, distribution, etc.) are also properties of the set of real numbers. It is only later in one section of the book where we make use of the "complex" nature of the complex numbers.