Math 232 Spring 2006 Project 04

Due February 17

Name

Be sure to re-read the WRITING GUIDELINES rubric, since it defines how your project will be graded. In particular, you may discuss this project with others but **you may not collaborate on the written exposition of the solution.**

"Obvious" is the most dangerous word in mathematics." – Eric Temple Bell

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Linear Independence

In class we showed

- If $\{w_1, w_2, w_3\}$ is a linearly independent set in $\mathbf{C}^{23}$, then the set

$$\{2w_1 + w_2 + 3w_3, -3w_1 + 2w_2 + 4w_3, w_1 - 3w_2 - 7w_3\}$$

  is linearly dependent.

- If $\{w_1, w_2, w_3\}$ is a linearly dependent set in $\mathbf{C}^{23}$, then the set

$$\{2w_1 + w_2 + 3w_3, -3w_1 + 2w_2 + 4w_3, w_1 + 2w_2 + 3w_3\}$$

  is linearly dependent.

Answer both of the following questions.

1. Suppose that $\{w_1, w_2, w_3\}$ is a linearly **independent** set in $\mathbf{C}^{23}$, Is the set

$$\{2w_1 + w_2 + 3w_3, -3w_1 + 2w_2 + 4w_3, w_1 + 2w_2 + 3w_3\}$$

   linearly independent?

2. Suppose that $\{w_1, w_2, w_3\}$ is a linearly **dependent** set in $\mathbf{C}^{23}$, Is the set

$$\{2w_1 + w_2 + 3w_3, -3w_1 + 2w_2 + 4w_3, w_1 - 3w_2 - 7w_3\}$$

   linearly independent?

You might find the following matrix information useful.

$$\begin{bmatrix} 2 & -3 & 1 \\ 1 & 2 & -3 \\ 3 & 4 & -7 \end{bmatrix} \xrightarrow{RREF} \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix} \text{ and } \begin{bmatrix} 2 & -3 & 1 \\ 1 & 2 & 2 \\ 3 & 4 & 3 \end{bmatrix} \xrightarrow{RREF} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$