Mathematics 121-A Exam 1 Spring 2006

February 2, 2006

Technology used:____ ____________________
Name

Directions:

- **Only write on one side of each page.**
- **Use terminology correctly.**
- **Show your work: No Work – No Credit**
- **Partial credit is awarded for correct approaches so justify your steps.**

Do any seven (7) of the following.

1. [10 points] A bicyclist's position is given by the function $s(t) = 3t - 2t^2$ where t is measured in seconds and $s(t)$ is measured in meters. The function $Q(h)$,below, gives the average velocity of the bicylist from time $t = 0.5$ to time $t = 0.5 + h$.

$$Q(h) = \frac{s(0.5+h) - s(0.5)}{h}.$$

Use your calculator to fill in the following table and then "guess" what the function Q should output for the input $h = 0$ (even though that input is **not** in the domain of Q).

h	0.1	0.001	0.00001
$Q(h)$	0.8	0.998	0.999 98

2. [10 points] Compute $\cot(\arcsin(x))$.

   (a) $\cot(\arcsin(x)) = \frac{\sqrt{1-x^2}}{x}$.

3. [10 points] Given $F(x) = \sqrt[3]{\tan(x^2+1)}$. Write down three functions f, g, h with the property that $(f \circ g \circ h)(x) = F(x)$.

   (a) $f(x) = \sqrt{x}$, $g(x) = \tan(x)$, $h(x) = x^2 + 1$.

4. [10 points] An open box with a square base is to be built for \$48.00. The sides of the box will cost \$3.00 per square foot and the base will cost \$4.00 per square foot. Express the volume of the box as a function of the length of one side of its (square) base.

   (a) Cost: $3(4xy) + 4(x^2) = 48$ dollars. So $y = \frac{48-4x^2}{12x}$. This gives a volume of $V(x) = x^2\frac{48-4x^2}{12x}$

5. [10 points] Write the equation of the circle whose graph is the result of moving the graph of $x^2 + y^2 - 4x + 8y - 5 = 0$ **two** units to the right and **three** units down.

   (a) Original circle is $(x-2)^2 + (y+4)^2 = 25$ so answer is $(x-4)^2 + (y+7)^2 = 5^2$

6. [10 points] Write the equation of the circle that has $(-3, 2)$ and $(5, -8)$ as the ends of a diameter.

(a) Center is at $\left(\frac{5-3}{2}, \frac{-8+2}{2}\right) = (1, -3)$. Radius is half a diameter: $\frac{1}{2}\sqrt{(5+3)^2 + (-8-2)^2} = \sqrt{41}$ The equation is $(x-1)^2 + (y+3)^2 = 41$.

7. [10 points] Find values for the constants c and δ so that the numbers x in the interval $-7 < x < -1$ are the same as the numbers x that satisfy the inequality $|x - c| < \delta$.

   (a) The interval is centered at $\frac{-7-1}{2} = -4$ and extends out 3 units from that center so $c = -4$ and $\delta = 3$: $|x - (-4)| < 3$.

8. [5 points each] Do both of the following.

   (a) Give an example of an even function that is not $f(x) = x^2$ or $g(x) = \cos(x)$ and use the symbolic definition (not a graph) to show it is even.

      i. $h(x) = x^2+1$ satisfies the property that for any input x, $h(-x) = (-x)^2+1 = x^2+1 = h(x)$. Thus $h(x) = x^2+1$ is an even function (its graph is symmetric with respect to the y - axis.)

   (b) Give an example of a function that is neither even nor odd and use the symbolic definition (not a graph) to show it fails to be odd.

      i. $h(x) = x^2 + x$ is neither even nor odd. It is not odd because it does not satisfy $h(-x) = -h(x)$ for all x. In particular, $h(-2) = 2 \neq -6 = -h(2)$. This function is also not even because it does not satisfy $h(-x) = h(x)$ for all x. In particular, $h(-2) = 2 \neq 6 = h(2)$.