Mathematics 221 Spring 2004 Project 4

Due: February 20

Collaborators **Name**

Directions: Be sure to follow the guidelines for writing up projects as specified in the course information sheet (passed out on the first day of class). Whenever appropriate, use in-line citations, including page numbers and people consulted when you present information obtained from discussion, a text, notes, or technology. **Only write on one side of each page.**

"*Never express yourself more clearly than you are able to think.*" — Niels Bohr

Project Description

For this project please submit your efforts on exactly one (1) of the following. (However, you should be able to do every problem in the list.)

1. Use components to explain why the following "Cross product rule" of Theorem 10.3 (page 645) is valid.

   If the vector functions $\overrightarrow{F}$ and $\overrightarrow{G}$ both have outputs in $\mathbf{R}^3$ and are differentiable at t, then

$$\left(\overrightarrow{F} \times \overrightarrow{G}\right)'(t) = \left(\overrightarrow{F'} \times \overrightarrow{G}\right)(t) + \left(\overrightarrow{F} \times \overrightarrow{G'}\right)(t).$$

2. (Arc length is independent of parametrization.) Suppose C is a curve parametrized by $\overrightarrow{R}_1(t) = \langle t, t^2, t^3 \rangle$, $5 \le t \le 10$. Suppose further that $\overrightarrow{R}_2(u) = \overrightarrow{R}_1(5u) = \left\langle 5u, (5u)^2, (5u)^3 \right\rangle$, $1 \le u \le 2$ is another parametrization obtained by replacing each occurrence of t with $g(u) = 5u$. It is easy to believe (and is true) that $\overrightarrow{R}_2$ is also a a parametrization of the curve C. Evidence supporting this claim is that the two arc lengths $\int_5^{10} \left\| \overrightarrow{R}\,'_1(t) \right\| \, dt$ and $\int_1^2 \left\| \overrightarrow{R}\,'_2(u) \right\| \, du$ turn out to be exactly the same number. But there is a more informative method of showing the equality of the integrals than using brute force to compute the integrals and compare the answers.

   Explain why $\int_{g(a)}^{g(b)} \left\| \overrightarrow{R}\,'_1(t) \right\| \, dt = \int_a^b \left\| \overrightarrow{R}\,'_2(u) \right\| \, du$ by using $\overrightarrow{R}_1(t) = \langle x(t), y(t), z(t) \rangle$, making the substitution $t = g(u)$ in the first integral, and showing the result is the second integral. You will need to use components in your argument.