

March 11, 2002

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Name

Directions: Be sure to include in-line citations, including page numbers if appropriate, every time you use the results of discussion, a text, notes, or technology. **Only write on one side of each page.**

*“Civilization advances by extending the number of important operations which we can perform without thinking of them.”* (Alfred North Whitehead)

## 1 Problems

1. Do **one** of the following.
  - (a) Determine the structure of the ring  $\mathbf{Z}[x]/(x^2 + 3, p)$  where
    - i.  $p = 3$
    - ii.  $p = 5$
  - (b) Fully describe the ring  $\mathbf{Z}[i]/(2 + i)$ .
2. Fully describe the ring obtained from  $\mathbf{Z}$  by adjoining an element  $\alpha$  satisfying the two relations  $2\alpha - 6 = 0$  and  $\alpha - 10 = 0$ .
3. Suppose we adjoin an element  $\alpha$  to  $\mathbf{R}$  satisfying the relation  $\alpha^2 = 1$ . Prove the resulting ring is isomorphic to the product ring  $\mathbf{R} \times \mathbf{R}$ , and find the element of  $\mathbf{R} \times \mathbf{R}$  which corresponds to  $\alpha$ .
4. Let  $\alpha$  denote the residue of  $x$  in the ring  $R' = \mathbf{Z}[x]/(x^4 + x^3 + x^2 + x + 1)$ . Compute the expressions for  $(\alpha^3 + \alpha^2 + \alpha)(\alpha + 1)$  and  $\alpha^5$  in terms of the basis  $(1, \alpha, \alpha^2, \alpha^3, \alpha^4)$ .
5. Do **one** of the following.
  - (a) In each case describe the ring obtained from  $\mathbf{Z}$  by adjoining an element  $\alpha$  satisfying the given relation.
    - i.  $\alpha^2 + \alpha + 1 = 0$
    - ii.  $\alpha^2 + 1 = 0$
  - (b) Let  $R = \mathbf{Z}/(10)$ . Determine the structure of the ring  $R'$  obtained from  $\mathbf{Z}$  by adjoining element  $\alpha$  satisfying each relation.
    - i.  $2\alpha - 6 = 0$
    - ii.  $2\alpha - 5 = 0$ .
6. Describe the ring obtained from  $\mathbf{Z}/12\mathbf{Z}$  by adjoining an inverse of 2. In particular, what ‘standard’ ring is isomorphic to  $\mathbf{Z}/12\mathbf{Z}$ ?