1 Math 434: Problem Set 4

1.1 Finite Abelian Groups

1. Find all the abelian groups of order less than or equal to 40 up to isomorphism.
2. Let G, H, K be finite abelian groups.
   (a) Prove if $G \times H \approx G \times K$ then $H \approx K$.
   (b) Give a counterexample to show the above cannot be true in general.
3. Do both of the following.
   (a) What is the smallest positive integer n such that there are exactly four nonisomorphic abelian groups of order n?
   (b) Show there are two abelian groups of order 108 that have exactly four subgroups of order 3.
4. Characterize those integers n such that the only abelian groups of order n are cyclic.
5. Determine the isomorphism class of $Aut\left(\mathbf{Z}_2 \times \mathbf{Z}_3 \times \mathbf{Z}_5\right)$.