1 Math 434: Problem Set 3

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1. Count how many distinct ways there are to color the faces of a cube with six colors if each color is used exactly once?

   (a) Up to a rotation, how many ways can the faces of a cube be colored three different colors?

2. (An 'applied' problem) Many large electronic circuits can be constructed by using smaller modules. The simplest such module is called a switching function and consists of a circuit that has n binary inputs and a single output. We convert this into a mathematical model by defining a **switching** or **Boolean function** of n variables to be any function $f : \mathbf{Z}_2^n \to \mathbf{Z}_2$.Note that since any switching function can have two possible values for each binary n-tuple and there are 2^n such binary n-tuples, then there are $2^{(2^n)}$ switching functions on n variables. Hence, with just 4 inputs, there are $2^{(2^4)} = 65\,536$ possible switching functions. But it is not necessary to construct all 65536 of these modules since many of them can be formed by merely permuting the input leads of another. For example, if f is a switching function on 3 binary variables a, b and c, then permuting the three leads yields another switching function g that satisfies
$$g(a,b,c) = f(b,c,a).$$
   We say two switching functions f and g are equivalent if g can be obtained from f by a permutation of the input variables. Thus, in the above example, g is equivalent to f via the permutation (a,c,b).

Example:
The $2^{(2^2)}$ switching functions on 2 binary variables are listed in the table below. Note the functions are subscripted by the decimal value equal to their outputs when the four outputs are read as a binary number.

Inputs		Outputs															
		f_0	f_1	f_2	f_3	f_4	f_5	f_6	f_7	f_8	f_9	f_{10}	f_{11}	f_{12}	f_{13}	f_{14}	f_{15}
0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1
0	1	0	0	0	0	1	1	1	1	0	0	0	0	1	1	1	1
1	0	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1
1	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1

Note: Under the single permutation (a,b) the above set of 16 possible switching functions reduces to 12 since
$f_2 \sim f_4$ and $f_3 \sim f_5$ and $f_{10} \sim f_{12}$ and $f_{11} \sim f_{13}$.
This corresponds to Burnside's Theorem since the group we are using $|G| = |\{(a),(a,b)\}| = 2$ and $S_{(a)} = G$ has 16 elements and $S_{a,b} = \{f_0, f_1, f_6 - f_9, f_{14}, f_{15}\}$ has 8 elements and
$$\frac{1}{2}[16+8] = 12.$$

How many switching functions on 3 (**three**) binary inputs if the group of permutations used in forming equivalent inputs is the full symmetric group on 3 elements?