February 20, 2001

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Name

SPRING SEMESTER 1997
EXAMINATION ONE
HONORS 213

Thompson 316, 3:00 P.M. Bryan Smith

> Ideologue, (noun). One who has an axiom to grind. -**L.Rollins**
> The beginning of wisdom is the definition of terms. -**Socrates**
> I think that one possible definition of our modern culture is that it is one in which nine-tenths of our intellectuals can't read any poetry. -**Randall Jarrell**

When taking the examination, be sure to give as complete answers as you can. You need not provide as much detail as that necessary in a homework assignment but your solutions should be complete and easy to follow.

Do any five of the following six problems. Be sure to clearly mark which of the problems you would like graded.

I 20 points

Let $\mathcal{M}$ be a finite projective plane. Prove that all lines in $\mathcal{M}$ have the same number of points lying on them.

II 20 points Using any previous result, prove the following portion of Pasch's Theorem.

If $\mathbf{A}, \mathbf{B}, \mathbf{C}$ are distinct noncollinear points and l is any line intersecting $\mathbf{AB}$ in a point between $\mathbf{A}$ and $\mathbf{B}$, then l also intersects either $\mathbf{AC}$ or $\mathbf{BC}$.

III 20 points

Using any result up to and including Proposition 2.1. prove the following.

There exist three distinct lines that are not concurrent.

IV 20 points

Exhibit a model of incidence geometry with exactly 6 points and 4 lines or prove that no such model exists.

V 20 points

Suppose M is an interpretation of "point", "line", "incident", and "between" that satisfies all of the incidence and betweenness axioms. Explain why if a statement S does not hold in M, then S cannot be proven using only the incidence and betweenness axioms.

VI 20 points

Suppose we add to the axioms of incidence geometry the following axioms:

1. The Euclidean parallel property.
2. The existence of only a finite number of points.
3. The existence of lines l and m such that the number of points lying on l is different from the number of points lying on m.

Show that this expanded axiom system is inconsistent.

[Hints:

1. Show that the first two items above imply the negation of the third.
2. Just focus on the case when there is a point $\mathbf{P}$ that is not incident with either l or m.]