Honors 213 Spring 2001 Problem Sheet 4

Due February 27, 2001

Name

Directions: Be sure to include in-line citations, including page numbers if appropriate, every time you use the results of discussion, a text, notes, or technology. **Only write on one side of each page.**

"*Reductio ad absurdum, which Euclid loved so much, is one of a mathematician's finest weapons. It is a far finer gambit than any chess play: a chess player may offer the sacrifice of a pawn or even a piece, but a mathematician offers the game.*" – Godfrey H. Hardy

Problems for Discussion in class (Not to be turned in)

1. Review Exercises page 103.
2. Exercise 29 page 109.

0.1 Outlined Problems: Do ONE of these.

1. Carefully present and justify each step of the following proof of Proposition 3.13 (c), (d). This is part of exercise 22 and exercise 23 on page 108.

   - (part c)
     (a) If $AB > CD$ then there is a point X satisfying $A * X * B$ and $AX \cong CD$.
     (b) But $CD \cong EF$ tells us $AX \cong EF$.
     (c) Thus X satisfies the defining characteristics of the claim $AB > EF$.
   - (part d)
     (a) $AB < CD$ so there is a point X with $AB \cong CX$ and $C * X * D$.
     (b) $CD < EF$ so there is a point Y with $CD \cong EY$ and $E * Y * F$.
     (c) $CD \cong EY$ and Proposition 3.12 implies there is a point X' satisfying $CX \cong EX'$ and $E * X' * Y$
     (d) $E * X' * Y$ and $E * Y * F$ and Proposition 3.3 implies $E * X' * F$.
     (e) SInce $AB \cong CX \cong EX'$ and $E * X' * F$ then $AB < EF$.

2. Exercise 269 on page 108. (Prove Proposition 3.17.) Carefully present and justify each step in the given proof.

0.2 Problems

1. Use Proposition 3.14.to deduce Proposition 3.15. (This is exercise 25 on page 109.)
2. Exercise 36 on page 119. Note that this tells us the 'obvious' fact that "every point interior to an angle is on a segment with one end on each of the rays forming the angle" is not provable using the Incidence and betweenness axioms.