Honors 213 **Spring 2001** **Problem Sheet 2**

January 26, 2001

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Name

Directions: Be sure to include in-line citations, including page numbers if appropriate, every time you use the results of discussion, a text, notes, or technology. **Only write on one side of each page.**

"*Mathematicians do not study objects, but relations among objects; they are indifferent to the replacement of objects by others as long as relations do not change. Matter is not important, only form interests them.*"
— Henri Poincaré

Discussion Problems

1. Chapter 2, Exercise 4: Negate the Euclidean parallel postulate.
2. Chapter 2, Exercise 7: For each pair of axioms of incidence geometry, construct an interpretation in which those two axioms are satisfied but the third axiom is not. (Thus, the axioms are *independent.*)

0.1 Outlined Problems (to be turned in with carefully written proofs)

1. Chapter 2, Exercise 10. (b): Prove or disprove. Two models of incidence geometry having exactly 4 points are isomorphic.
2. Chapter 2, Major exercise 1 :
   (a) Let M be a projective plane. Define a new interpretation M' by taking as "points" of M' the lines of M and as "lines" of M' the points of M, with the same incidence relation. Prove that M' is also a projective plane.
   (b) Suppose further that M has only finitely many points. Prove all the lines of M have the same number of points lying on them.

0.2 Problems (to be turned in with proofs that your audience will find easy to follow)

1. Chapter 2, Exercise 11: Construct a model of incidence geometry gthat has neither the elliptic, hyperbolic or Euclidean parallel properties. (The hint in the text is referring to the fact that each of these properties is stated using universal quantifiers ($\forall$). So the problem is to devise a model for which three existentially quantified statements all hold.)
2. Chapter 2, Major exercise 7. (c) . Let M be a finite projective plane so that, according to Major Exercise 1, all lines in M have the same number of points lying on them; call this number $n+1$. Prove the following:
   (a) Each point in M has $n+1$ lines passing through it. (Outlined in class – not to be written up.)
   (b) The total number of points in M is n^2+n+1. (Outlined in class – not to be written up.)
   (c) The total number of lines in M is n^2+n+1.