1 Honors 213 - A Spring 2001

1.1 Exam 1

February 16, 2001

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Name

Textbook/Notes used: ____

Directions: Be sure to include in-line citations, including page numbers if appropriate, every time you use a text or notes or technology. **Only write on one side of each page.**

The Problems

1. Using any previous results, prove the following portion of Proposition 3.8.

   If D is in the interior of angle $\angle CAB$; then:

   (a)

   (b) no point on the opposite ray to ray $\overrightarrow{AD}$ is in the interior of angle $\angle CAB$.

2. Using any previous results, prove the **Crossbar Theorem**. If ray $\overrightarrow{AD}$ is between ray $\overrightarrow{AB}$ and ray $\overrightarrow{AC}$, then $\overrightarrow{AD}$ intersects segment BC.

3. Do **one** of the following.

   (a) Using any previous result, prove Proposition 2.4. For every point there is at least one line not passing through it.

   (b) Use a truth table to determine if the logical statement $((P \wedge\ \tilde{}Q) \wedge (R \wedge\ \tilde{}R)) \Rightarrow P \Rightarrow Q$ is a tautology. (This statement encodes the method of proof by contradiction.) You will need 8 horizontal lines to encompass all possible truth values of P, Q, and R.

4. Modelling problem: The following picture represents an interpretation of the undefined terms “point”, “line”, and “between” for which all incidence and betweenness axioms hold. The given circle is a circle in the Euclidean plane. ‘Points’ are the Euclidean points **inside** the circle, ‘lines’ are the portions of Euclidean lines that are inside the circle (that is, the chords of the circle), and a point C is “between” two other points if all three are on the same chord of the circle and C is between the other two in the Euclidean sense. Use this model and the standard Euclidean model to carefully explain why transitivity of parallelism is independent of the axioms of incidence and betweenness. That is, give a full explanation of why the following statement is independent of the axioms.

   Given distinct lines l, m, and n.If l is parallel to m and m is parallel to n, then l is parallel to n.

Figure 1: