Cylindrical and Spherical Coordinates

Extra Homework Exercises

1. Convert each equation to cylindrical coordinates and sketch its graph in $\mathbf{R}^3$.

   (a) $z = x^2 + y^2$

   (b) $z = x^2 - y^2$

   (c) $\frac{x^2}{4} - \frac{y^2}{9} + z^2 = 0$

2. Convert each equation to spherical coordinates and sketch its graph in $\mathbf{R}^3$.

   (a) $z^2 = x^2 + y^2$

   (b) $4z = x^2 + 3y^2$

   (c) $x^2 + y^2 - 4z^2 = 1$

3. Convert each equation to rectangular coordinates and sketch its graph in $\mathbf{R}^3$.

   (a) $z = r^2 \sin(2\theta)$

   (b) $r = \sin(\theta)$

   (c) $\rho^2 \sin^2(\phi) = 1$

   (d) $\rho^2 \sin(\phi)\cos(\phi)\cos(\theta) = 1$

4. A solid lies above the cone $z = \sqrt{x^2 + y^2}$ and below the sphere $x^2 + y^2 + z^2 = z$.

   (a) Write a description of the solid in terms of inequalities involving cylindrical coordinates.

   (b) Write a description of the solid in terms of inequalities involving spherical coordinates.

5. The point (x, y, z) lies on an ellipsoid if (in the following R, a, b, c are all constants)

$$\begin{aligned} x &= aR\sin(\phi)\cos(\theta) \\ y &= bR\sin(\phi)\sin(\theta) \\ z &= cR\cos(\phi) \end{aligned}$$

   Find an equation for this ellipsoid in rectangular coordinates.

6. What is the area in xy −space corresponding to the area of the region in $r\theta$ − space given by

$$\{(r, \theta) :\ r_0 \le r \le r_0 + \Delta r,\ \theta_0 \le \theta \le \theta_0 + \Delta\theta?$$