1 Mathematics 433 Fall 2000 Problems to Turn in: 4

September 7, 2000

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Name

Directions: Be sure to include in-line citations, including page numbers if appropriate, every time you use the results of discussion, a text, notes, or technology. **Only write on one side of each page.**
"Mathematicians do not study objects, but relations among objects; they are indifferent to the replacement of objects by others as long as relations do not change. Matter is not important, only form interests them."
— Henri Poincaré

Problems

1. **You must do this problem.**
   (a) Prove the set $Aut(G)$of all automorphisms of a group G forms a group, the binary operation being the composition of functions.
   (b) Determine the group of automorphisms of each of the following groups.
      i. $(Z, +)$ (also known as Z^+)
      ii. A cyclic group of order 10.
      iii. S_3
2. Prove $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$ are conjugate elements in $GL(2, R)$ but are not conjugate in $SL(2, R)$.
3. Do **one** of the following.
   (a) Describe all homomorphisms $\phi : (Z, +) \to (Z, +)$. Determine which are one-to-one, which are onto and which are isomorphisms.
   (b) Do all of the following.
      i. Prove that if a group contains exactly one element of order 2, then that element is in the center of the group.
      ii. Suppose $\phi : G \to G'$ is an onto homomorphism. Prove, if G is cyclic, then G' is cyclic.
      iii. Suppose $\phi : G \to G'$ is an onto homomorphism. Prove, if G is abelian, then G' is abelian.
4. Do either of the following.
   (a) Find all subgroups of S_3 and determine which of these are normal.
   (b) Find all subgroups of the quaternion group and determine which of these are normal.
5. Do either of the following.
   (a) Prove by giving an explicit example that $GL(2, \mathbf{R})$ is not a normal subgroup of $GL(2, \mathbf{C})$.
   (b) Let $\phi : G \to G'$ be an onto homomorphism and let N be a normal subgroup of G.
      i. Show that the set $\phi(N) = \{\phi(n) : n \in N\}$ is a subgroup of G'.
      ii. Prove that $\phi(N)$ is a normal subgroup of G'.