1 Mathematics 433 Fall 2001 Problems to Turn in: 3

September 14, 2001

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Name

Directions: Be sure to include in-line citations, including page numbers if appropriate, every time you use the results of discussion, a text, notes, or technology. **Only write on one side of each page.**
" *You don't understand anything until you learn it more than one way.*" – Marvin Minsky

Problems

1. You **must** do this problem.
   (a) If H is a subgroup of G, then by the **centralizer,** $C(H)$, of H we mean the set $\{x \in G : xh = hx$ for all h
   Prove that $C(H)$ is a subgroup of G.
   (b) Must the centralizer of an element of a group be Abelian?
   (c) Must the center of a group be Abelian?

2. Do one (1) of the following.
   (a) Suppose that G is a group of order 16 and that, by direct computation, you know that G has at least nine elements x such that $x^8 = e$.
      i. Can you conclude that G is not cyclic?
      ii. What if G has at least five elements x such that $x^4 = e$?
      iii. Generalize your results as a reasonable conjecture.
   (b) If G is an Abelian group and contains cyclic subgroups of orders 4 and 5, what other sizes of cyclic subgroups **must** G contain?

3. Do all of the following.
   (a) Let $b' = aba^{-1}$.Prove that $(b')^n = ab^n a^{-1}$
   (b) Prove if $aba^{-1} = b^2$,then $a^3ba^{-3} = b^8$.
   (c) Prove that the map $\phi : GL(n, R) \to GL(n, R)$ defined by $\phi(A) = (A^t)^{-1}$ is an automorphism.

4. Do:
   (a) Let H be a subgroup of G and let $g \in G$. The **conjugate subgroup** gHg^{-1} of G is defined to be the set of all conjugates ghg^{-1} where $h \in H$. Prove that $gHg^{-1}is$ a subgroup of G.
   (b) Prove that a subgroup H of G is normal in G if and only if $gHg^{-1} = H$ for all $g \in G$.