1 Additional Exercises: Symmetry of Plane Figures

1. Prove the set of symmetries of a figure F in the plane form a group.

2. List all symmetries of

   (a) a square

   (b) a regular pentagon

3. List all symmetries of the following figures

   (a) Figure 1.4

   (b) Figure 1.5

   (c) Figure 1.6

   (d) Figure 1.7

4. Compute the fixed point of $t_a\rho_\theta$ algebraically.

5. Explicitly verify the rules:

   (a) $t_a t_b = t_{a+b}$

   (b) $\rho_\theta \rho_\eta = \rho_{\theta+\eta}$

   (c) $rr = i$

   (d) $\rho_\theta t_a = t_{a'}\rho_\theta$, where $a' = \rho_\theta (a)$

   (e) $rt_a = t'_a r$, where $a' = r(a)$

   (f) $r\rho_\theta = \rho_{-\theta} r$.

6. Prove that O is not a normal subgroup of M.

7. Let SM denore the subset of orientation-preserving motions of the plane. Prove SM is a normal subgroup of M and determine its index in M.

8. Prove the map $\phi : M \to \{i, r\}$ given by $\phi (t_a\rho_\theta) = i$ and $\phi (t_a\rho_\theta r) = r$ is a homomorphism.

9. Compute the effect of a rotation of the axes through an angle η on the expressions $t_a\rho_\theta$ and $t_a\rho_\theta r$ for a motion.

10. Find an isomorphism from the group SM *to* the subgroup of $GL(2, \mathbf{C})$ of matrices of the form $\begin{bmatrix} a & b \\ 0 & 1 \end{bmatrix}$ with $|a| = 1$.

11. Do

    (a) Write the formulas for teh motions t_a, ρ_θ and r it tems of the complex variables $z = x+iy$.

    (b) Show every motion has the form $m(z) = \alpha z + \beta$ or $m(z) = \alpha \overline{z} + \beta$, where α, β are complex numbers with $|\alpha| = 1$.