1 Additional Problems

1.1 Operations on Subsets

1. Let S be the set of subsets of order 2 of the dihedral group D_3.Determine the orbits for the action of D_3 on S by conjugation.

2. Determine the orbits for left multiplication and for conjugation on the set of subsets of order 3 of D_3.

3. Let U be a subset of a finite group G, and suppose $|U|$ and $|G|$ have no common factor. Is the stabilizer of U trivial for the operation of conjugation?

4. Consider the operation of left multiplication by G on the set of its subsets. Let U be a subset whose orbit $\{gU : g \in G\}$ partitions G. Let H be the unique subset in this orbit which contains the identity e of G. Prove H is a subgroup of G and the sets gU are the left cosets of H.

5. Let S be a finite set on which a group G operates transitively (that is, there is only one orbit). Let U be a subset of S. Prove the subsets gU cover S evenly. That is, every element of S is in the same number of sets gU.

1.2 Sylow Theorems

1. How many elements of order 5 are contained in a group of order 20?

2. Prove no group of order pq, where p and q are prime, is simple.

3. Find Sylow 2 - subgroups in the following cases:

   (a) D_{10}
   (b) T (the group of rotational symmetries of the regular tetrahedron.)
   (c) O (the group of rotational symmetries of the cube.)
   (d) I (the group of rotational symmetries of the regular dodecahedron.)

4. Let G be a group of order $p^l m$. Prove G contains a subgroup of order p^r for every integer $1 \leq r \leq l$.