1 Additional Exercises: Cosets

- Determine the index $[Z : nZ]$.
- Prove directly that distinct cosets do not overlap.
- Prove every group whose order is a power of a prime p contains an element of order p.
- Give an example showing that left cosets and right cosets of $GL(2, R)$in GL$(2, C)$ are not always equal.
- Let H, K be subgroups of a group G of orders $3, 5$ respectively. Prove $H \cap K = \{e\}$.
- Do
  1. Let G be an abelian group of odd order. Prove the map $\phi : G \to G$ defined by $\phi(x) = x^2$ is an automorphism.
  2. Generalize the above result.
- Let W be additive subgroup of R^m of solutions of a system of homogeneous linear equations $AX = \overrightarrow{0}$. Show the solutions of a non-homogeneous system $AX = B$ form a coset of W.
- Do:
  1. Prove that every subgroup of index 2 is normal.
  2. Give an example of a subgroup of index 3 that is not normal.
- Let G, H be the following subgroups of $GL(2, R)$:

$$G = \left\{ \begin{bmatrix} x & y \\ 0 & 1 \end{bmatrix} \right\}, \quad H = \left\{ \begin{bmatrix} x & 0 \\ 0 & 1 \end{bmatrix} \right\}, \; x > 0.$$

  An element of G can be represented by a point in the (x, y) plane. Draw the partitions of the plane into left and into right cosets of H.