Mathematics 300-A **Spring 2002**

Exam 2

April 12, 2002

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Name

Directions: Be sure to include in-line citations, including page numbers if appropriate, every time you use a text or notes or technology.

Only write on one side of each page.

"I never know how much of what I say is true." — Bette Midler

The Problems

Do any two (2) of the following

1. Using any result through the corollary to Theorem 4.4 and exercise 26 of chapter 4, show that if one pair of sides of quadrilateral $\square \mathbf{A}BDC$ satisfies the definition of a convex quadrilateral, then so does the other pair of sides.

2. Using any result previous to Theorem 6.6 and exercise 1 of chapter 6,do the following. Suppose lines l and l' have a common perpendicular MM'. Let points A and B be on l so that they do not have M as a midpoint. Prove A and B are not equidistant from l'.

3. List statments equivalent in neutral geometry to Hilbert's parallel property. (± 1 point each.)

Do any two (2) of the following

1. Using any result through chapter 4 but no exercises from that chapter, show that statement $S_{4.12}$ is equivalent to Hilbert's parallel postulate. Statement $S_{4.12}$ is "In hyperbolic geometry, if l, m, n are distinct lines, $l \,||\, m$ and $m \,||\, n$ then $l \,||\, n$." [Note: this is an 'if and only if' problem so there are two things to show.]

2. Using any result previous to Proposition 4.3 and exercise 12 of chapter 4,as well as the existence of the midpoints of segments, prove that every segment has a unique midpoint.

3. Using any result through exercise 13 in chapter 6 do the following. In Theorem 4.1 it was proved in neutral geometry that if alternate interior angles formed by a transversal t to lines l, m are congruent, then the lines l and m are parallel. Strengthen this result in hyperbolic geometry by proving the following.

   In hyperbolic geometry, if alternate interior angles formed by a transversal t to lines l, m are congruent, then the lines l and m are **divergently** parallel. [Hint: Let M be the midpoint of the segment PQ of the transversal. Here, P, Q are the points of intersection with l and m, respectively.]

4. Using any material from chapter 6, do the following. Let P denote the Euclidean parallel postulate and H denote the hyperbolic parallel axiom. Show that any statement S in the language of neutral geometry that is a theorem in Euclidean geometry ($P \Longrightarrow S$) and whose negation is a theorem in hyperbolic geometry ($H \Longrightarrow \tilde{}S$) is equivalent (in neutral geometry) to the parallel postulate. [This is a slick way to find statements that are equivalent to the parallel postulate.]