Mathematics 300-A — Spring 2002

Exam 2

April 12, 2002

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Name

Directions: Be sure to include in-line citations, including page numbers if appropriate, every time you use a text or notes or technology.
Only write on one side of each page.

The Problems

1. Do any two (2) of the following.

   (a) Using any result up to and including Proposition 3.4, prove **Pasch's Theorem.** If A, B, C are distinct non collinear points and l is any line intersecting segment AB in a point between A and B, then l also intersects either segment AC or segment BC. If C does not lie on l, then l does not intersect both AC and BC.

   (b) Using any previous results, prove the following portion of Proposition 3.8. If D is in the interior of angle $\sphericalangle CAB$; then:

   i. no point on the opposite ray to ray $\overrightarrow{AD}$ is in the interior of angle $\sphericalangle CAB$.

   (c) Using any previous results, prove the **Crossbar Theorem**. If ray $\overrightarrow{AD}$ is between ray $\overrightarrow{AB}$ and ray $\overrightarrow{AC}$, then $\overrightarrow{AD}$ intersects segment BC.

2. Do any three (3) of the following.

   (a) Using any previous result, including the first part of the proposition, prove the second half of Proposition 3.3 :
   Given $A * B * C$ and $A * C * D$, then $A * B * D$.

   (b) Recall that a well-formed-statement (wfs) is **independent** of an axiomatic system if neither that statement nor its negation can be deduced from the axioms. Prove the following (wfs) is independent of the axioms of incidence geometry.
   "For any two lines l and m there exists a one-to-one correspondence between the set of points incident with line l and the set of points incident with line m. "

   (c) Using any result up to and including Proposition 3.19, prove Proposition 3.20.(Angle Subtraction)
   Given ray $\overrightarrow{BG}$ between ray $\overrightarrow{BA}$ and ray $\overrightarrow{BC}$, ray $\overrightarrow{EH}$ between ray $\overrightarrow{ED}$ and ray $\overrightarrow{EF}$, $\sphericalangle CBG \cong \sphericalangle FEH$, and $\sphericalangle ABC \cong \sphericalangle DEF$. Then $\sphericalangle GBA \cong \sphericalangle HED$.

   (d) Using any result up to and including part (a) of Proposition 3.9, prove the following. If D is a point interior to triangle ΔABC, then any ray emanating from D must intersect one of the sides of the triangle.

   (e) Using any previous result, prove part (d) of Proposition 3.13. If $AB < CD$, and $CD < EF$, then $AB < EF$.