Smith **Math 290** **Spring 2013**

Final Exam **May 17, 2013**

I affirm this work abides by the university's Academic Honesty Policy.

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Print Name, then Sign

Directions:

- **Only write on one side of each page.**
- **Use terminology correctly.**
- **Partial credit is awarded for correct approaches so justify your steps.**

Do BOTH of these "Computational" problems

C.1. [15 points] Let V be a subspace of $\mathbf{C}^n$ and define $V^{\perp} = \{\vec{x} \in \mathbf{C}^n \mid \langle \vec{x}, \vec{v} \rangle = 0 \text{ for every } \vec{v} \in V\}$. Show that $V^{\perp}$ is a subspace of $\mathbf{C}^n$.

C.2. [15 points] Let $T : P_2 \longrightarrow P_3$ be given by $T(p) = x^3 p'' - x^2 p' + 3p$. Find the matrix representation of T with respect to the bases B, C where $B = \left\{x+1, x+2, x^2\right\}$ and $C = \left\{1, x, x^2, x^3\right\}$.

Do any two (2) of these "In Class, Text, or Homework" problems

M.1. [15 Points] Prove that if $T : U \to V$ is a linear transformation and W is a subspace of U then the image of W under T, $T(W) = \{T(\vec{u}) : \vec{u} \in W\}$, is a subspace of V.

M.2. [15 Points] Prove Theorem SMEE (Similar Matrices have Equal Eigenvalues)

1. Suppose A and B are similar matrices. Then the characteristic polynomials of A and B are equal, that is, $p_A(x) = p_B(x)$.

M.3. [15 Points] Prove Theorem SSRLT, Spanning Set for Range of a Linear Transformation

1. Suppose that $T : U \to V$ is a linear transformation and $S = \{u_1, u_2, u_3, ..., u_t\}$ spans U. Then $R = \{T(u_1), T(u_2), T(u_3), ..., T(u_t)\}$ spans $R(T)$.

(more problems on reverse side)

Do three (3) of these "Other" problems

T.1. [15 Points] Property DVA in the definition of a vector space (see the **Useful Information** below) tells us that scalar multiplication distributes across the sum of **two** vectors. Use the principle of mathematical induction to prove the following fact that Professor Beezer used repeatedly throughout his book.

1. **Theorem:** Suppose V is a vector space, n is any positive integer, α is a scalar, and $\vec{v}_1, \vec{v}_2, \vec{v}_3, \cdots, \vec{v}_n$ are vectors in V. Then $\alpha \sum_{k=1}^{n} \vec{v}_k = \sum_{k=1}^{n} \alpha \vec{v}_k$.

T.2. [15 Points] Suppose that $T : V \longrightarrow V$ is a linear transformation. Prove that $(T \circ T)(\vec{v}) = \vec{0}$ for every $\vec{v} \in V$ if and only if $R(T) \subseteq K(T)$ (the range of T is a subset of the kernel of T).

T.3. [15 Points] Given $A \in M_{m,n}$ and $B \in M_{n,m}$, show that if $\vec{x} \in N(AB - I_m)$, then $B\vec{x} \in N(BA - I_n)$

T.4. [15 points] Let $B = \{1, x, x^2\}$ be the standard basis for P_2 and $g(x) = -1 + 2x$ a fixed vector in P_2. Find a basis for the subspace $V = \{f \in P_2 \mid \langle \rho_B(f), \rho_B(g) \rangle = 0\}$.[Note that $\langle \vec{x}, \vec{y} \rangle$ denotes the inner product of the vectors $\vec{x}, \vec{y} \in \mathbf{C}^3$.]

Extra Credit [10 points] Given $A \in M_{m,n}$ and $B \in M_{n,m}$, show that if $N(BA - I_n) = \left\{\vec{0}_n\right\}$ then $N(AB - I_m) = \left\{\vec{0}_m\right\}$.

Useful Information

- **Property DVA of the definition of a vector space** V.

  If $\alpha \in \mathbf{C}$ and $\mathbf{u}, \mathbf{v} \in V$, then $\alpha(\mathbf{u} + \mathbf{v}) = \alpha\mathbf{u} + \boldsymbol{\alpha}\mathbf{v}$.