Smith **Math 290** **Spring 2013**

Exam 4

I affirm this work abides by the university's Academic Honesty Policy.

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Print Name, then Sign

Directions:

- **Only write on one side of each page.**
- **Partial credit is awarded for correct approaches so justify your steps.**

Do one (1) of these "Computational" problems

C.1. [10, 10 points] The sets $B = \left\{1+x, 2+x^2, 3+x+x^2\right\}$ and $C = \left\{3, 2-x, 1-x^2\right\}$ are bases for P_2.

1. Determine the change of basis matrix $C_{B,C}$.
2. Use it to compute $\rho_C\left(5\left(1+x\right)+4\left(2+x^2\right)\right)$.

C.2. [15, 5 points] Given the linear transformation $T : P_2 \to C^3$ defined by

$$T\left(a+bx+cx^2\right) = \begin{bmatrix} 2a+3b-c \\ 2b-2c \\ a-b+2c \end{bmatrix}$$

1. Find a basis for the Range of T.
2. Find a vector in $\mathbf{C}^3$ that is not in the range of T.

Do two (2) of these "In Class, Text, or Homework" problems

1. [20 points] Prove Theorem ILTLT: Suppose that $T : U \longrightarrow V$ is an invertible linear transformation. Then the function $T^{-1} : V \longrightarrow U$ is a linear transformation.
2. [20 points] Prove that a function $T : U \to V$ is a linear transformation if and only if for all scalars c, d and for all vectors $\overrightarrow{x}$, $\overrightarrow{y} \in U$ we have $T\left(c\overrightarrow{x} + d\overrightarrow{y}\right) = c\,T\left(\overrightarrow{x}\right) + d\,T\left(\overrightarrow{y}\right)$.
3. [20 points] Prove Theorem ILTLI : Suppose that $T : U \longrightarrow V$ is an injective linear transformation and $S = \{u_1, u_2, u_3, \ldots, u_t\}$ is a linearly independent subset of U. Then $R = \{T(u_1), T(u_2), T(u_3), \ldots, T(u_t)\}$ is a linearly independent subset of V.

Do any two (2) of these "Other" problems

1. [20 Points] Prove that if X is a subspace of V and $T : U \to V$ is a linear transformation, then the preimage of X, $T^{-1}\left(X\right) = \left\{\vec{u} \in U : T\left(\vec{u}\right) \in X\right\}$, is a subspace of U.
2. [20 Points] Suppose $T : U \longrightarrow V$ is a linear transformation that satisfies $T \circ T = T$.Prove that 0 and 1 are the only eigenvalues of T.
3. [14, 6 Points] Suppose U, V, and W are vector spaces and $T : U \longrightarrow V$, $S : V \longrightarrow W$ are linear transformations.

(a) Prove $\ker(T) \subseteq \ker(S \circ T)$. [Here, $\ker(T)$ denotes the kernel of T.]

(b) If B,C, D are bases of U, V, W respectively, what result about the matrices $M_{B,C}^{T}$, $M_{C,D}^{S}$ and $M_{B,D}^{S\circ T}$ corresponds to $\ker(T) \subseteq \ker(S \circ T)$?

Useful information

1. $M_{B,C}^{T} = [\rho_C(T(\vec{u}_1)) \mid \rho_C(T(\vec{u}_2)) \mid \cdots \mid \rho_C(T(\vec{u}_n))]$ and If T is the identity map, then $M_{B,C}^{T}$ is the change of basis matrix $C_{B,C}$