Smith **Math 290** **Fall 2008**

Exam 4 **December** 4, 2008

I affirm this work abides by the university's Academic Honesty Policy.

____
Print Name, then Sign

Directions:

- **Only write on one side of each page.**
- **Use terminology correctly.**
- **Partial credit is awarded for correct approaches so justify your steps.**

Define all of the following.

D.1. [8 points] Linear Transformation $T : U \to V$ (where U and V are vector spaces).

D.2. [8 points] Surjective linear transformation $T : U \to V$

D.3. [9 points] The coordinate transformation ρ_B where $B = \left\{\vec{b}_1, \cdots, \vec{b}_n\right\}$ is a basis for the vector space V.

Do one (1) of these "Computational" problems

C.1. [8, 7 points] Professor Beezer has proven a theorem that tells us the subset $S = \{1 + x, 2 - x + x^2, 3 + x^3, 1 + 2x^2$ · of P_3 is linearly independent if and only if the corresponding set of coordinate vectors relative to the standard basis of P_3 are linearly independent in $\mathbf{C}^4$.

1. Compute those coordinate vectors and show the set of them is linearly independent in $\mathbf{C}^4$.
2. Compute the coordinate vector of x^3 relative to the basis $B = \{1 + x, 2 - x + x^2, 3 + x^3, 1 + 2x^2 + 3x^3\}$ of P_4.

C.2. [15 points] Find a basis for the range of the linear transformation $T : P_2 \to C^3$ defined by $T(f) = \begin{bmatrix} f(0) \\ f'(1) \\ f(2) \end{bmatrix}$.

Do any two (2) of these "In Class, Text, or Homework" problems

M.1. [15 Points] Let V be a vector space with dimension n and $B = \left\{\vec{b}_1, \cdots, \vec{b}_n\right\}$ a basis for V. Prove Theorem VRLT from the textbook.

Theorem: The function $\rho_B : V \to \mathbf{C}^n$ is a linear transformation.

M.2. [15 Points] Prove Theorem ILVLT from the textbook.

Theorem: Suppose that $T : U \to V$ is an invertible linear transformation. Then the function $T^{-1} : V \to U$ is a linear transformation.

M.3. [15 Points] Prove Theorem CSLTS from the textbook. Suppose that $T : U \to V$ and $S : V \to W$ are both surjective linear transformations. Then $(S \circ T) : U \to W$ is a surjective linear transformation. You may use, without proving it, the fact that $S \circ T$ is a linear transformation.

Do two (2) of these "Other" problems

T.1. [15 Points] Prove the following.

If $T : U \to V$ is a linear transformation and X is a subspace of V, then $T^{-1}(X) = \{\vec{u} \in U : \ T(\vec{u}) \in X\}$ is a subspace of U.

T.2. [15 Points] Suppose $T : U \to V$ is a function that satisfies the single condition $T(\alpha\vec{x} + \vec{y}) = \alpha T(\vec{x}) + T(\vec{y})$ for every $\vec{x}, \vec{y}$ in U and every α in $\mathbf{C}$. Prove that T must be a linear transformation.

T.3. [15 Points] Let B be any fixed invertible matrix in M_{nn}.It can then be shown (but you don't have to show it) that the function $T : M_{nn} \to M_{nn}$ defined by $T(A) = B^{-1}AB$ is a linear transformation. Determine, with proof, whether or not this function T is

1. Injective
2. Surjective.